

Coulomb matrix elements

$$\Phi_a(\vec{r}) = \frac{u_{n_a l_a}(r)}{r} Y_{l_a m_a}(\vartheta, \varphi)$$

$$\int d^3 r \int d^3 r' \overline{\Phi_4(\vec{r}) \Phi_3(\vec{r}')} \frac{1}{|\vec{r} - \vec{r}'|} \Phi_2(\vec{r}') \Phi_1(\vec{r})$$

$$= \sum_{k=0}^{\infty} \int dr \overline{u_4(r)} u_1(r) \int dr' \overline{u_3(r')} u_2(r') \frac{r_{<}^k}{r_{>}^{k+1}}$$

$$\times \frac{4\pi}{2k+1} \sum_{\mu=-k}^k \int d\hat{r} \overline{Y_4(\hat{r}) Y_{k\mu}(\hat{r})} Y_1(\hat{r}) \int d\hat{r}' \overline{Y_3(\hat{r}') Y_{k\mu}(\hat{r}')} Y_1(\hat{r}')$$

Gaunt coefficients

$l=1$

$$k=0 \quad \frac{1}{\sqrt{4\pi}}$$

$$k=2 \quad \frac{1}{\sqrt{20\pi}} \begin{pmatrix} -1 & +\sqrt{3} & -\sqrt{6} \\ -\sqrt{3} & 2 & -\sqrt{3} \\ -\sqrt{6} & +\sqrt{3} & -1 \end{pmatrix}$$

$l=2$

$$k=0 \quad \frac{1}{\sqrt{4\pi}}$$

$$k=2 \quad \frac{1}{\sqrt{196\pi}} \begin{pmatrix} -\sqrt{20} & +\sqrt{30} & -\sqrt{20} & 0 & 0 \\ -\sqrt{30} & \sqrt{5} & +\sqrt{5} & -\sqrt{30} & 0 \\ -\sqrt{20} & -\sqrt{5} & \sqrt{20} & -\sqrt{5} & -\sqrt{20} \\ 0 & -\sqrt{30} & +\sqrt{5} & \sqrt{5} & -\sqrt{30} \\ 0 & 0 & -\sqrt{20} & +\sqrt{30} & -\sqrt{20} \end{pmatrix}$$

$$k=4 \quad \frac{1}{\sqrt{196\pi}} \begin{pmatrix} 1 & -\sqrt{5} & \sqrt{15} & -\sqrt{35} & \sqrt{70} \\ +\sqrt{5} & -4 & +\sqrt{30} & -\sqrt{40} & +\sqrt{35} \\ \sqrt{15} & -\sqrt{30} & 6 & -\sqrt{30} & \sqrt{15} \\ +\sqrt{35} & -\sqrt{40} & +\sqrt{30} & -4 & +\sqrt{5} \\ \sqrt{70} & -\sqrt{35} & \sqrt{15} & -\sqrt{5} & 1 \end{pmatrix}$$