

**Exercise Sheet 10** due 16 July 20181. *Gaunt coefficients*

Verify the Gaunt coefficients  $\langle l, m | k, m - m' | l, m' \rangle$  for  $p$  and  $d$ -shells in the basis of spherical harmonics:

$$l = 1 \quad k = 0 \quad \frac{1}{\sqrt{4\pi}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$k = 2 \quad \frac{1}{\sqrt{20\pi}} \begin{pmatrix} -1 & +\sqrt{3} & -\sqrt{6} \\ -\sqrt{3} & 2 & -\sqrt{3} \\ -\sqrt{6} & +\sqrt{3} & -1 \end{pmatrix}$$

$$l = 2 \quad k = 0 \quad \frac{1}{\sqrt{4\pi}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$k = 2 \quad \frac{1}{\sqrt{196\pi}} \begin{pmatrix} -\sqrt{20} & +\sqrt{30} & -\sqrt{20} & 0 & 0 \\ -\sqrt{30} & \sqrt{5} & +\sqrt{5} & -\sqrt{30} & 0 \\ -\sqrt{20} & -\sqrt{5} & \sqrt{20} & -\sqrt{5} & -\sqrt{20} \\ 0 & -\sqrt{30} & +\sqrt{5} & \sqrt{5} & -\sqrt{30} \\ 0 & 0 & -\sqrt{20} & +\sqrt{30} & -\sqrt{20} \end{pmatrix}$$

$$k = 4 \quad \frac{1}{\sqrt{196\pi}} \begin{pmatrix} 1 & -\sqrt{5} & \sqrt{15} & -\sqrt{35} & \sqrt{70} \\ +\sqrt{5} & -4 & +\sqrt{30} & -\sqrt{40} & +\sqrt{35} \\ \sqrt{15} & -\sqrt{30} & 6 & -\sqrt{30} & \sqrt{15} \\ +\sqrt{35} & -\sqrt{40} & +\sqrt{30} & -4 & +\sqrt{5} \\ \sqrt{70} & -\sqrt{35} & \sqrt{15} & -\sqrt{5} & 1 \end{pmatrix}$$

Hint: For  $\langle l, 0 | k, 0 | l, 0 \rangle$  perform the integral over the Legendre polynomials, for the other matrix elements, use the ladder operators.

Show that for  $k > 0$  the trace  $\sum_m \langle l, m | k, 0 | l, m \rangle$  vanishes.