time-dependent perturbation theory

$$\hat{H} = \hat{H}_0 + \hat{H}_p(t) \qquad\qquad \hat{H}_0|\psi_n\rangle = E_n|\psi_n\rangle$$

ansatz $$|\Psi\rangle = \sum_n a_n(t)\exp(-iE_nt/\hbar)\,|\psi_n\rangle$$

order-by-order expansion

$$\frac{d}{dt}a_q^{(p+1)}(t) = \frac{1}{i\hbar}\sum_n a_n^{(p)}(t)\exp(-i(E_q - E_n)t/\hbar)\,\langle\psi_q|\hat{H}_p(t)|\psi_n\rangle$$

Fermi's Golden Rule

harmonic perturbation $$\hat{H}_p(t) = \hat{H}_{po}\Big(\exp(-i\omega t) + \exp(i\omega t)\Big)$$

transition rate $$|\psi_m\rangle \to |\psi_j\rangle$$

$$w_{jm} = \frac{2\pi}{\hbar}\,|\langle\psi_j|\hat{H}_{po}|\psi_m\rangle|^2\,\delta(E_{jm} - \hbar\omega)$$

approximation to Dirac delta function

$$\int_{-L/2}^{L/2} dz\, e^{i(k_n-k_m)z} = \frac{1}{L}\,\frac{sin((k_n-k_m)\,L/2)}{(k_n-k_m)\,L/2} = \frac{1}{L}\,\delta_{n,m} \qquad (k_n = 2\pi n/L)$$

$$\int_{\infty}^{\infty} dz\, e^{i(k-k')z} = \lim_{L\to\infty} \frac{2sin((k-k')\,L/2)}{(k-k')} = 2\pi\,\delta(k-k')$$