Hilbert space and Dirac notation

wave function (state vector): $|\psi\rangle$ ”ket”

adjoint (transpose & complex conjugate): $\langle\psi| = |\psi\rangle^{\dagger}$ “bra”

inner product: $\langle\psi|\varphi\rangle = \langle\varphi|\psi\rangle^{*}$

norm: $|\psi|^2 = \langle\psi|\psi\rangle$ real (probability interpretation!)

Hilbert space: vector space with inner product

expansion in ortho-normal basis $|\psi_n\rangle$

state vector: $|f\rangle = \sum |\psi_n\rangle \langle\psi_n|f\rangle$

linear Operator: $A = \sum |\psi_n\rangle \langle\psi_n|A|\psi_m\rangle \langle\psi_m|$

identity operator: $I = \sum |\psi_n\rangle \langle\psi_n|$

Trace: $\mathrm{Tr}\, A = \sum \langle\psi_n|A|\psi_n\rangle$ (independent of basis)

Hilbert space and Dirac notation

unitary operator: $U^\dagger = U^{-1}$ *or* $U^\dagger U = I$ (leaves inner products unchanged)

basis transformation: $|f_{new}\rangle = U\,|f_{old}\rangle$ and $A_{new} = U A_{old} U^\dagger$ (physics unchanged)
changing both, state vectors and operators, leaves physics unchanged.

only changing state vector but not the operators changes the physical state
example: time evolution
this is called the **Schrödinger picture**
alternatively we could only change the operators and keep the state vector fixed,
this is called the **Heisenberg picture**

Hermitian operator: $M^\dagger = M$
real eigenvalues (observable)
eigenvectors with different eigenvalues are orthogonal