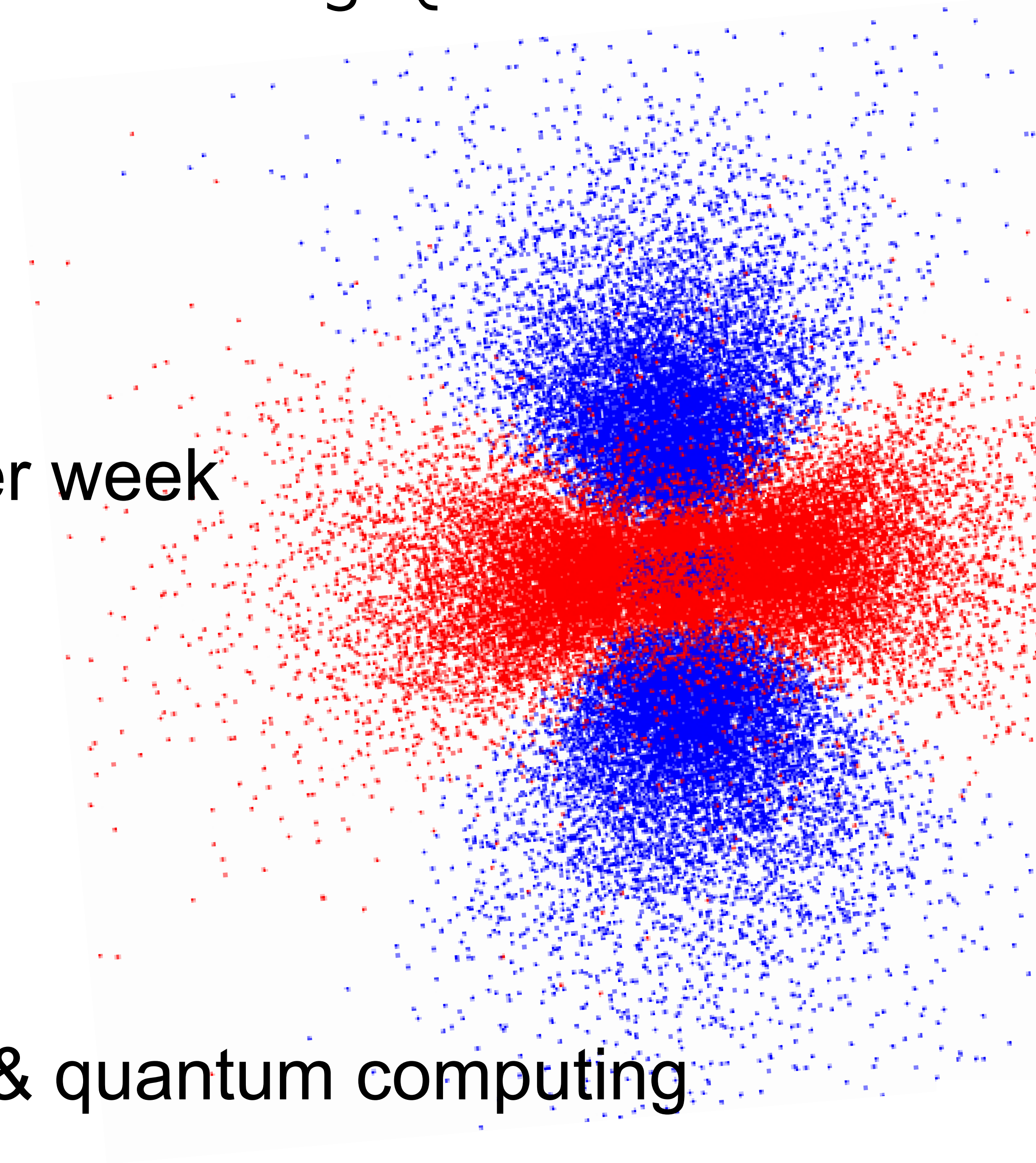
Applied Quantum Mechanics

course information: `www.cond-mat.de/teaching/QM`



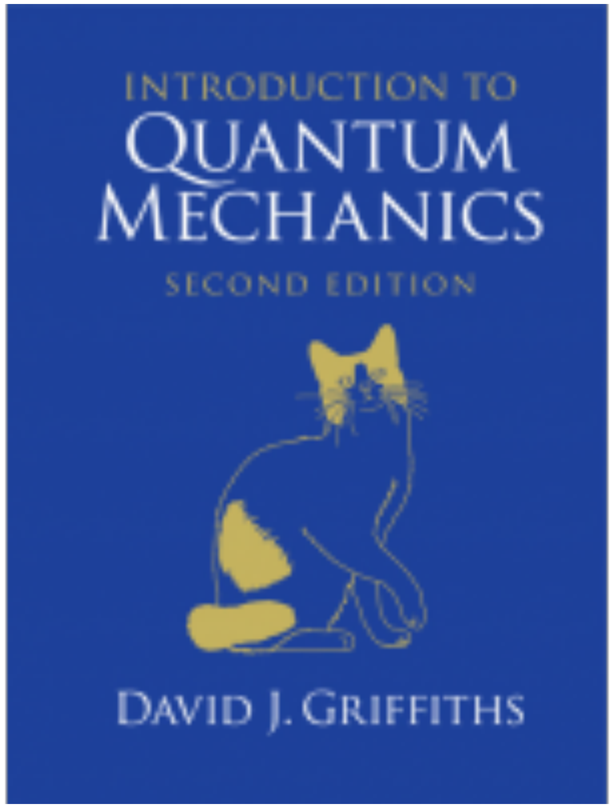
course policies:

- exercises:
  - hand in before lecture
  - solve at least 50% of problems per week
  - attend tutorials
  - prepare to explain your solution
    or where you got stuck

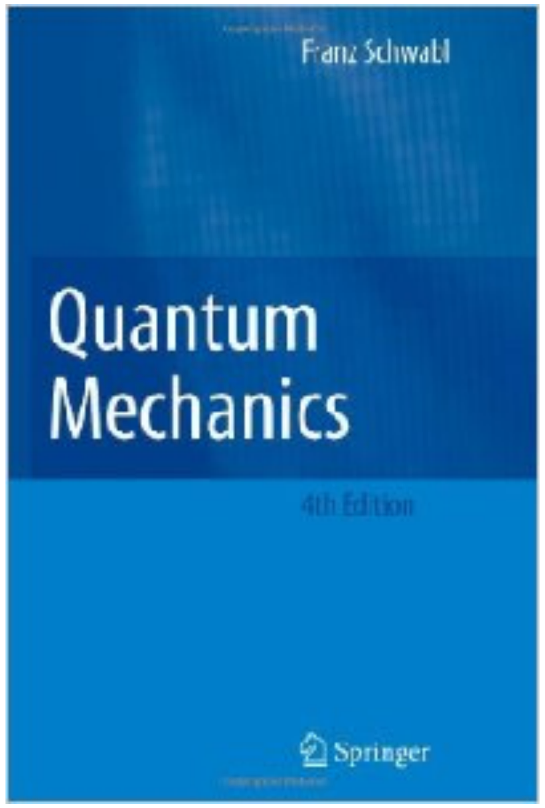
course goals:

- ideas of QM: chemistry/electronics & quantum computing
- simple simulations: python/Jupyter
- mathematical techniques

textbooks



D.J. Griffiths:
Introduction to
Quantum Mechanics
Cambridge Univ. Press



F. Schwabl:
Quantum Mechanics

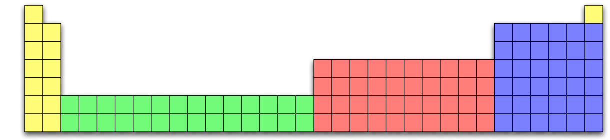
Springer

Matter is made of atoms

If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generations of creatures, what statement would contain the most information in the fewest words? I believe it is the *atomic hypothesis* (or the atomic *fact*, or whatever you wish to call it) that all things are made of atoms – little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is and *enormous* amount of information about the world, if just a little imagination and thinking are applied.

Lecture 1 of The Feynman Lectures on Physics, Vol. I (1961)

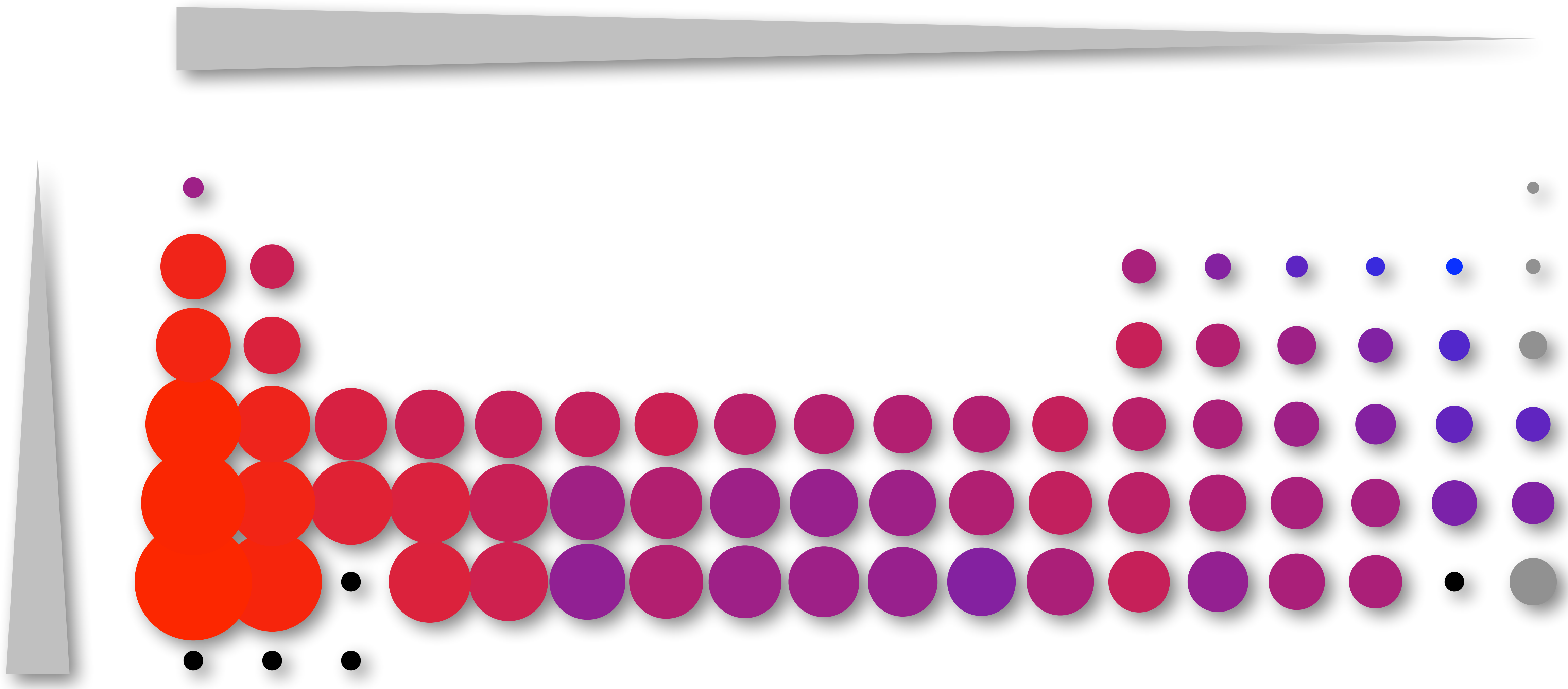
periodic table



H																	He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Lr	Rf	Db	Sg	Bh	Hs	Mt									

La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No

atomic radii



typical size 10^{-10} m = 1 Å

exercise

given
N_e electrons, N_i atomic nuclei of mass M_α und charge Z_α,
solve:

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r}_1,\ldots,\vec{r}_{N_e};\vec{R}_1,\ldots,\vec{R}_{N_i};t) = H\,\Psi(\vec{r}_1,\ldots,\vec{r}_{N_e};\vec{R}_1,\ldots,\vec{R}_{N_i};t)$$

$$H = -\frac{\hbar^2}{2m}\sum_{j=1}^{N_e}\nabla_j^2 - \sum_{\alpha=1}^{N_i}\frac{\hbar^2}{2M_\alpha}\nabla_\alpha^2 - \frac{1}{4\pi\epsilon_0}\sum_{j=1}^{N_e}\sum_{\alpha=1}^{N_i}\frac{Z_\alpha e^2}{|r_j - R_\alpha|} + \frac{1}{4\pi\epsilon_0}\sum_{j<k}^{N_e}\frac{e^2}{|r_j - r_k|} + \frac{1}{4\pi\epsilon_0}\sum_{\alpha<\beta}^{N_i}\frac{Z_\alpha Z_\beta e^2}{|R_\alpha - R_\beta|}$$

The underlying laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known, and the difficulty is only that exact applications of these laws lead to equations which are too complicated to be soluble. It therefore becomes desirable that approximate practical methods of applying quantum mechanics should be developed, which can lead to an explanation of the main features of complex atomic systems without too much computation.

P.M.A Dirac, *Proceedings of the Royal Society* **A123**, 714 (1929)

why quantum mechanics?

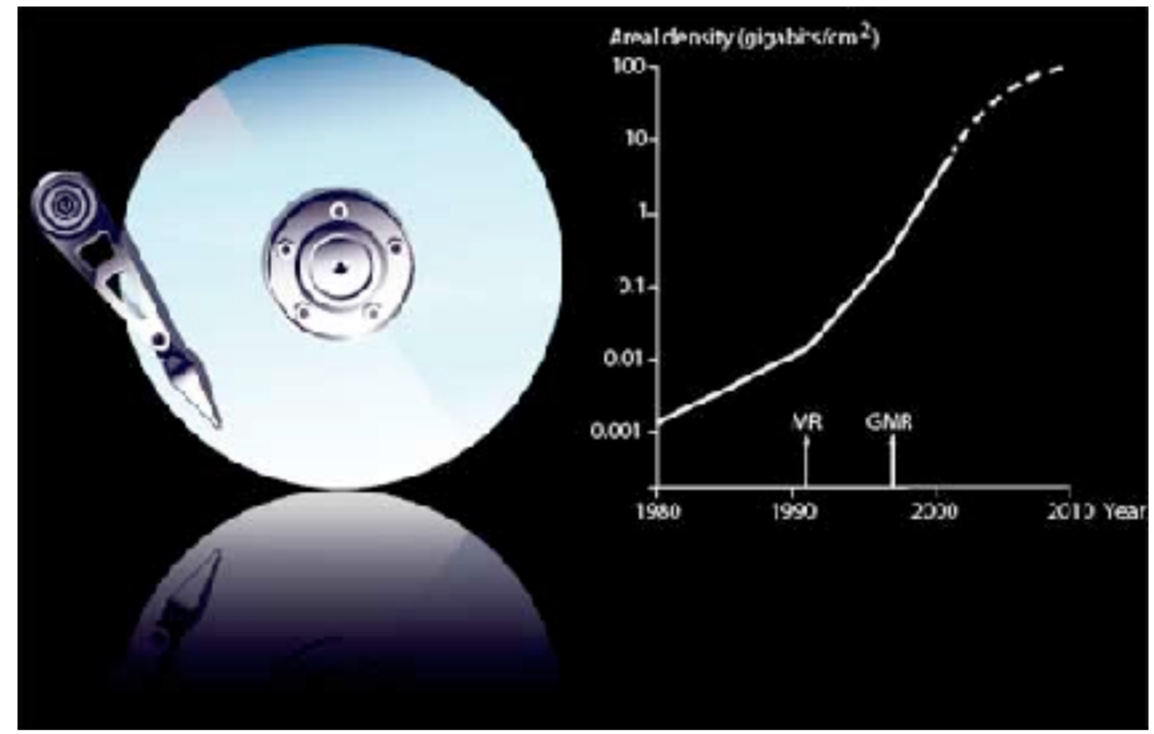
matter is made from atoms
why do atoms exist?
what are their properties?
how do they interact?
how do they assemble into matter?

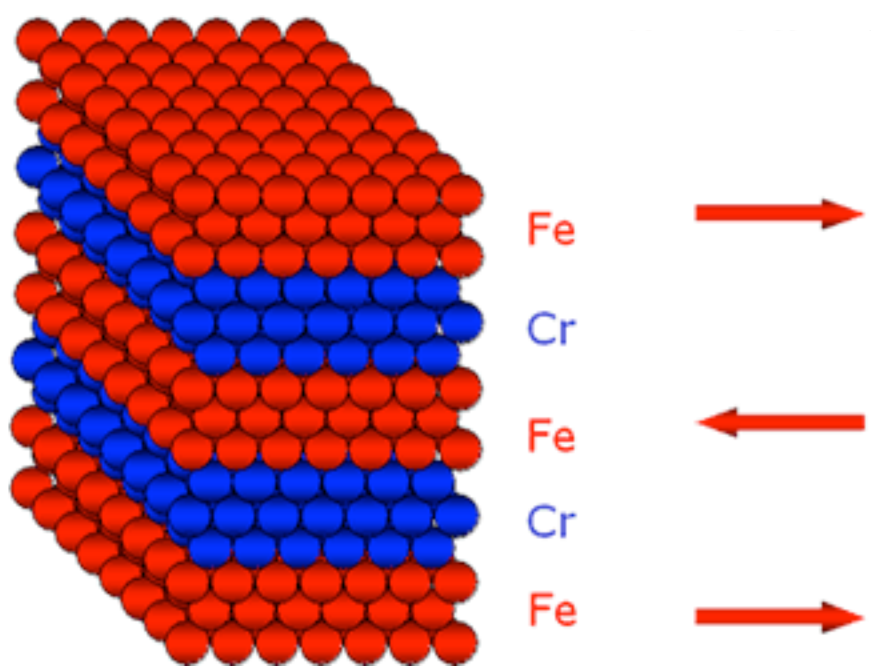
quantum science:
chemistry (bonding of (few) atoms)
condensed-matter physics ~ 10^{23} atoms

quantum engineering:
electronics/IT (transistor, GMR)
optics (laser: communication, materials tooling)
medicine (MRI, CT, PET)
energy (solar cells, nuclear power)
metrology (atomic clocks/GPS)
nanosystems (STM, 10 nm process)
quantum computing

giant magnetoresistance

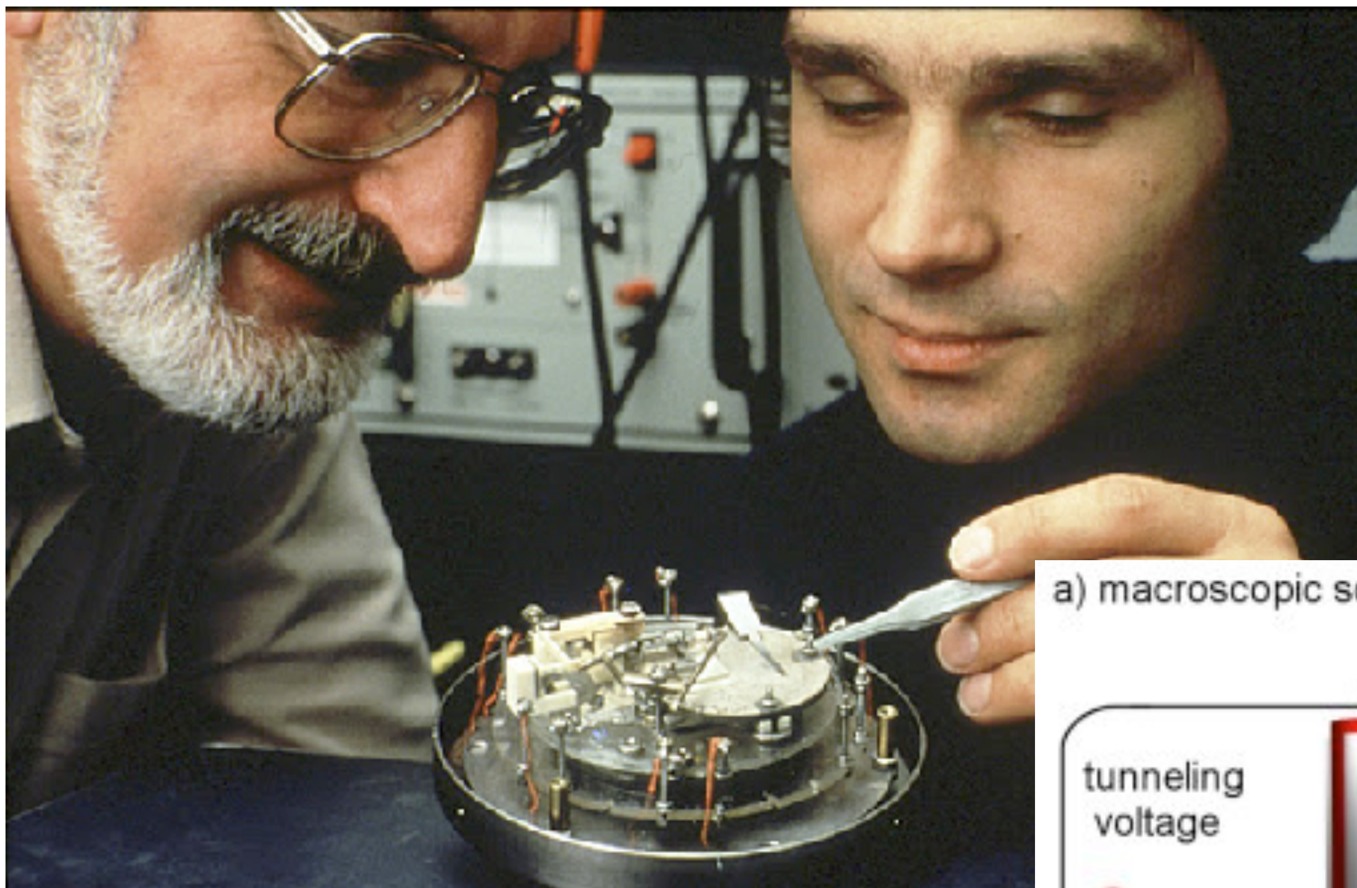
Peter Grünberg (Jülich) and Albert Fert (Paris), 1988
Nobel prize in Physics 2007

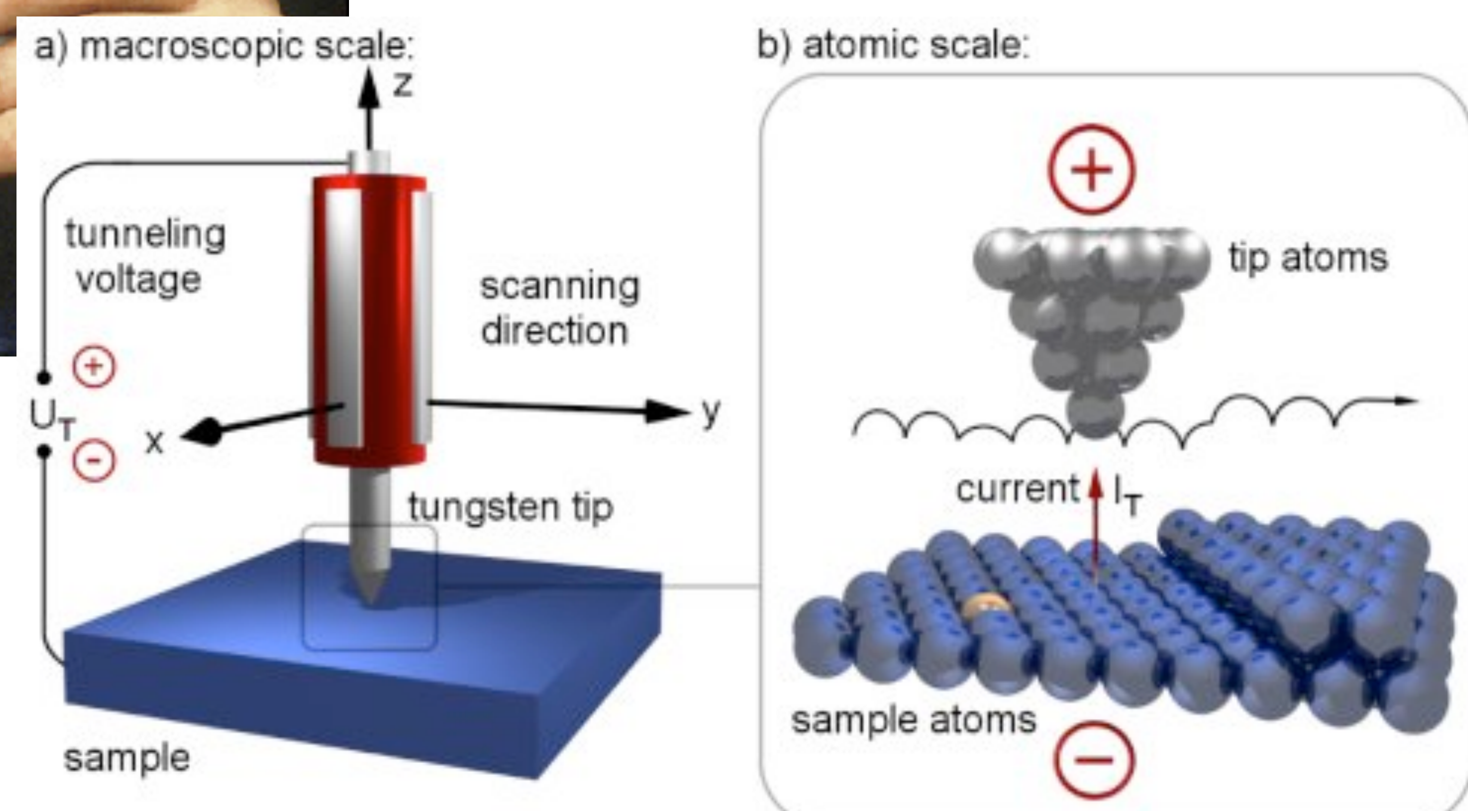




scanning tunneling microscope

Gerd Binnig and Heinrich Rohrer, IBM Rüschlikon, 1981
Nobel Prize in Physics 1986



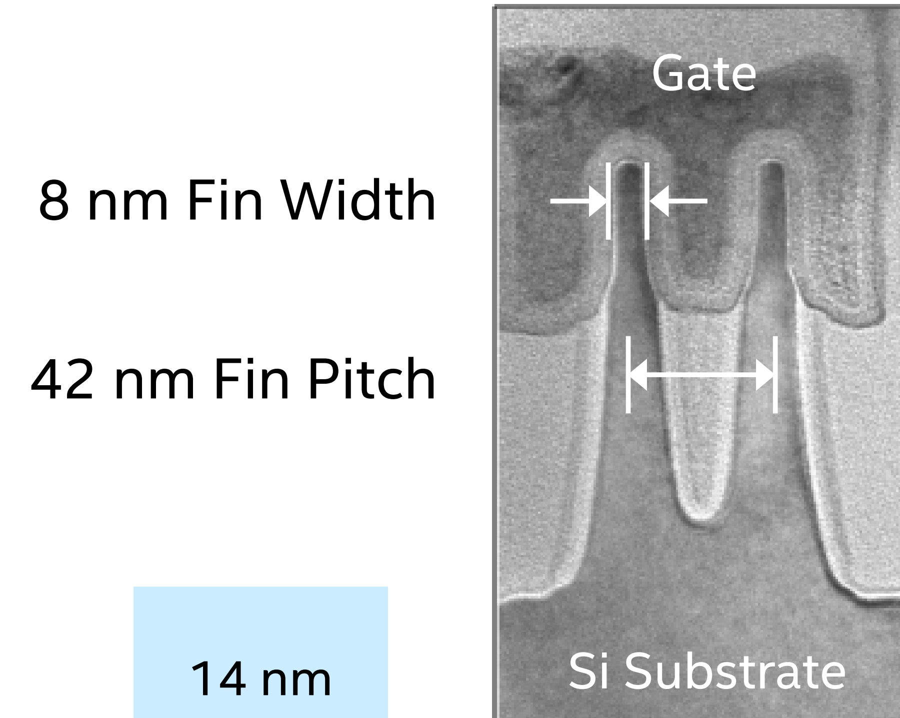


scanning tunneling microscope

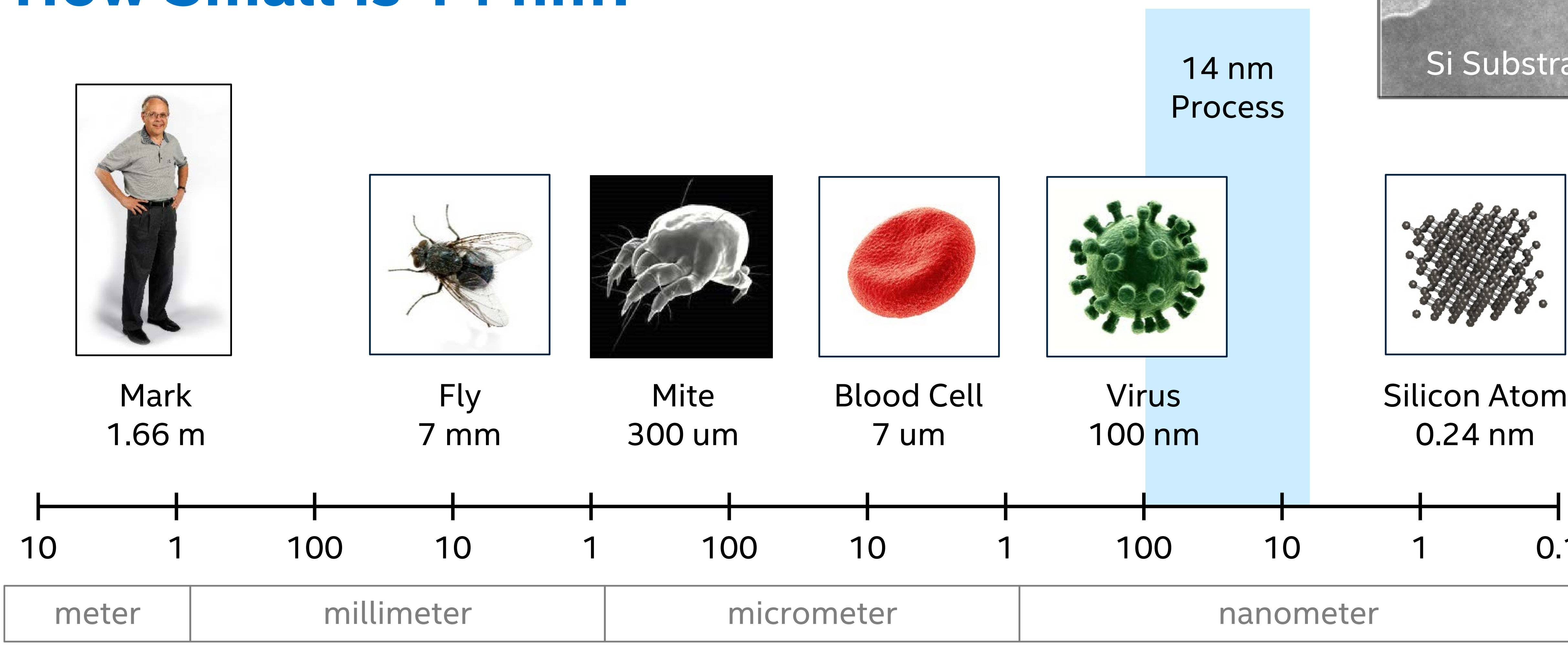
seeing and manipulating atoms

Microelectronics: 14nm process

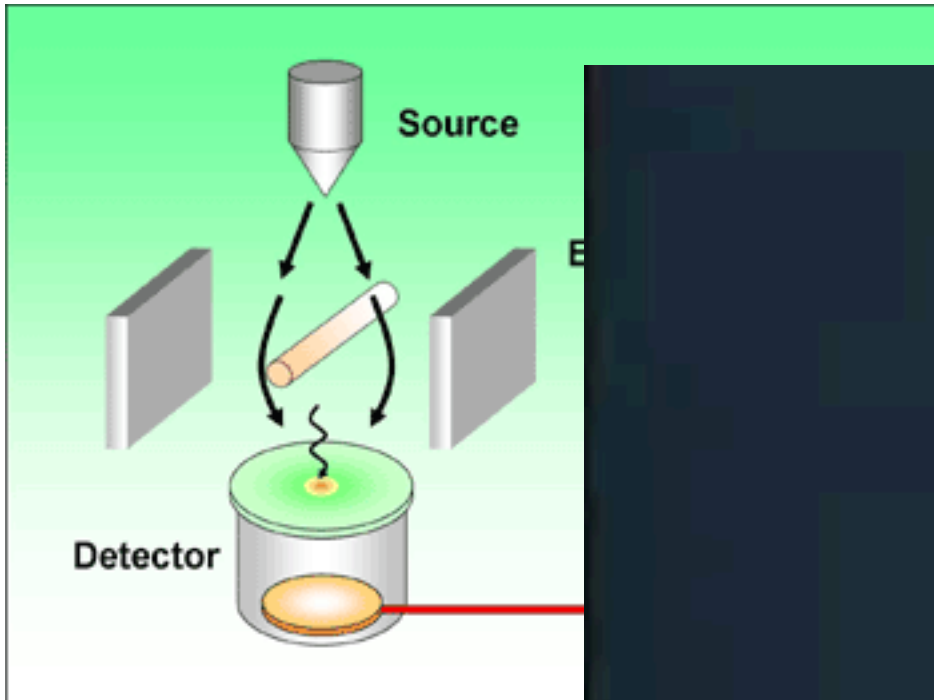
14 nm Tri-gate Transistor Fins



How Small is 14 nm?



double-slit experiment with electrons



http://www.hitachi.com/rd/research/em/doubleslit.html

see also: R.P. Feynman: Feynman Lectures on Physics, Vol. 3
Ch. 1: Quantum Behavior

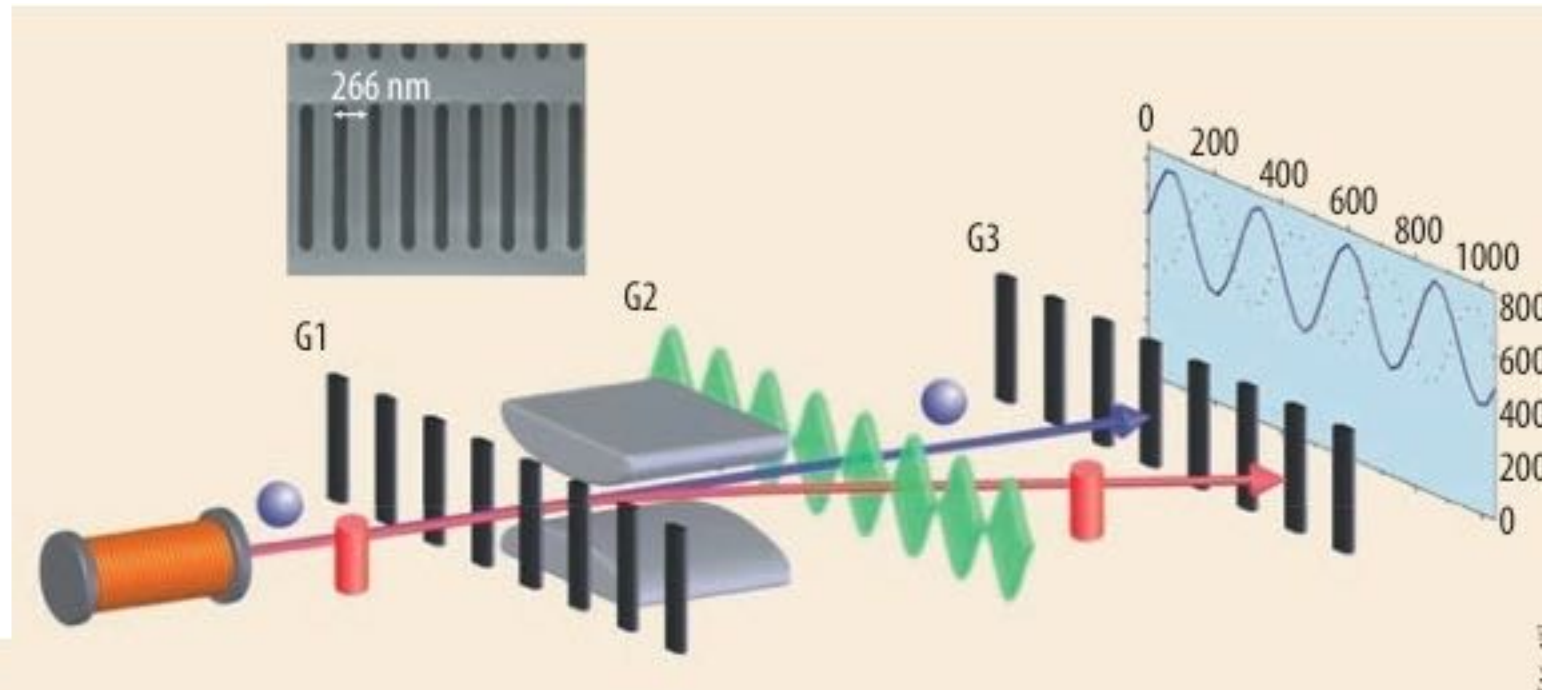
Interferometrie mit komplexen Molekülen

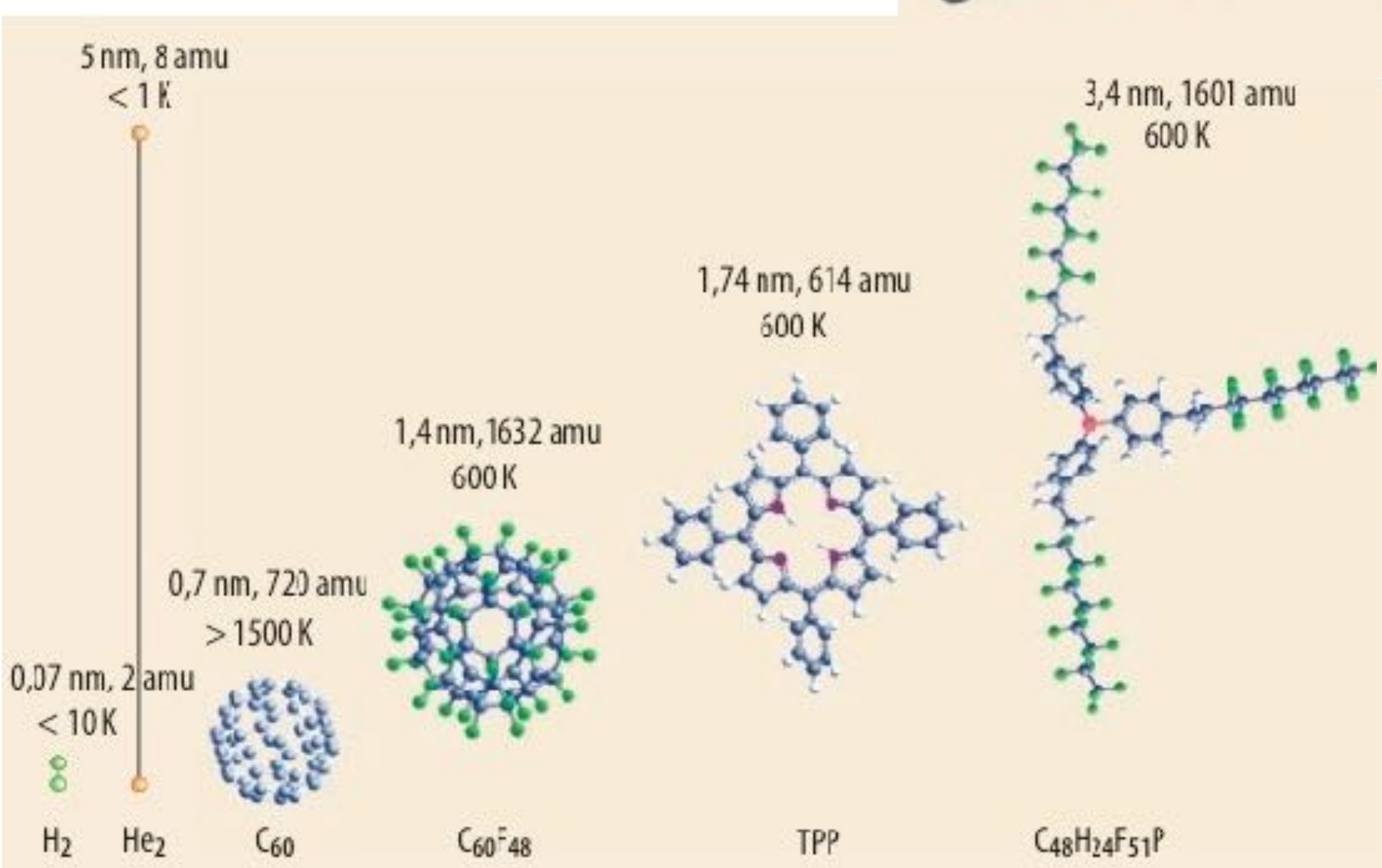
Physik Journal **9** Okt. 2010, p. 37

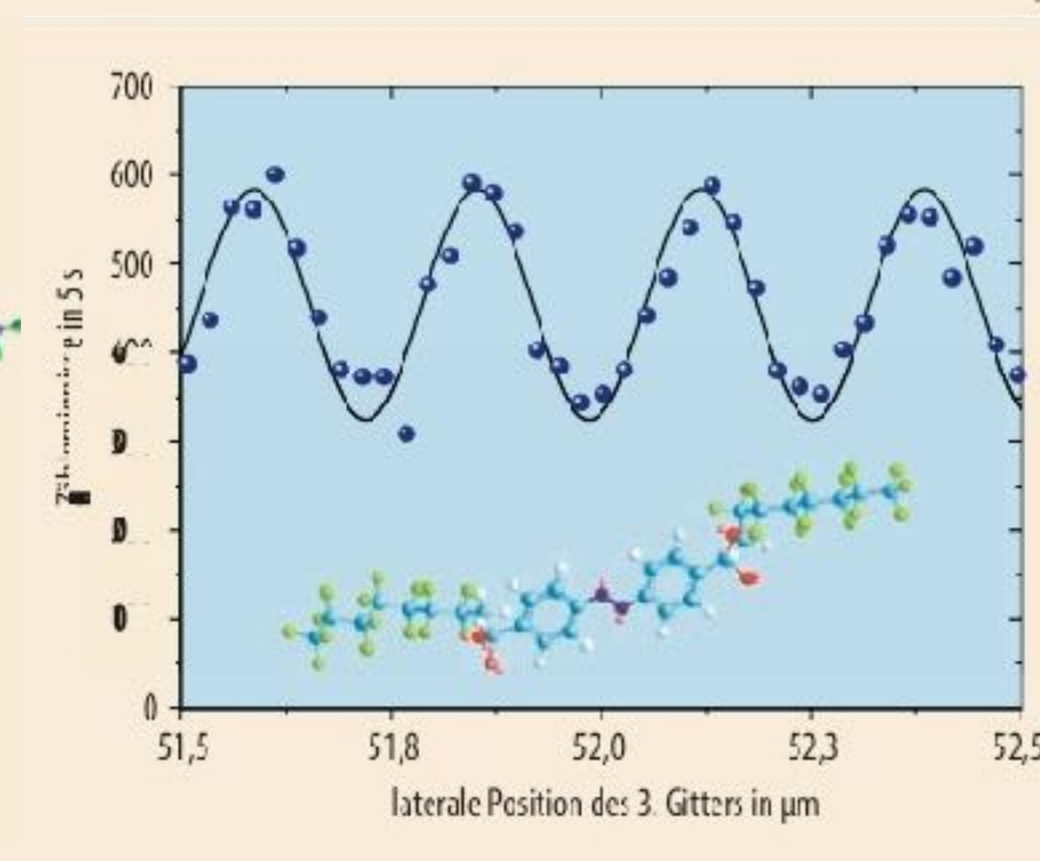
Wie man Einblick in das Innenleben von quantenmechanisch delokalisierten Molekülen gewinnt

Markus Arndt, Stefan Gerlich, Klaus Hornberger und Marcel Mayor

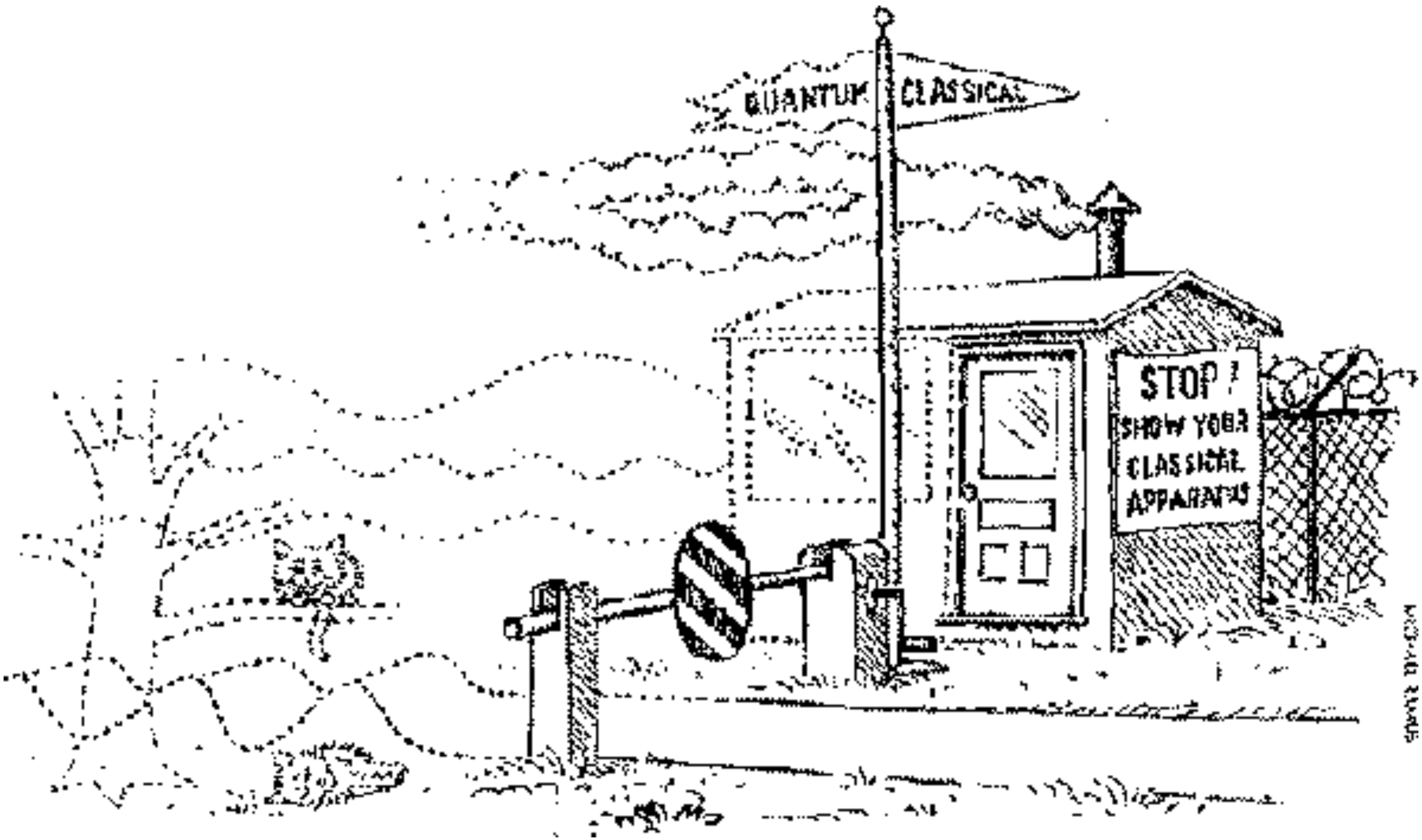
not just electrons behave as waves ...







quantum vs. classical behavior



quantum mechanics

unusual concepts:

particle-wave duality (Bohr)

uncertainty relations (Heisenberg)

probability interpretation (Born)

superposition (strict linearity) (Schrödinger cat)

entanglement (Einstein/EPR)

decoherence & measurements

Feynman: I think I can safely say that nobody understands quantum mechanics

Mermin: Shut up and calculate ⇒ **do the exercises!**

time-dependent Schödinger equation

particle		wave	dispersion relation		solution $e^{i(kx-\omega t)}$
p	$=$	$\hbar k$			
E	$=$	$\hbar\omega$	photons	$\omega = ck$	$\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = \frac{\partial^2\Phi}{\partial x^2}$
			electrons	$E = \frac{p^2}{2m}$	$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$

time-dependent Schrödinger equation

$$i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r},t)\right)\Psi(\vec{r},t)$$

1st derivative:
complex waves,
initial-value problem:

$$\Psi(\vec{r},t+\delta t) \approx \Psi(\vec{r},t) + \frac{\partial\Psi(\vec{r},t)}{\partial t}\,\delta t$$

Born interpretation of wave function:

$|\psi(r,t)|^2$ is probability density of finding electron at time *t* in position *r*

$\psi(r,t)$ probability amplitude (**normalize**!?)

exercise

given
N_e electrons, N_i atomic nuclei of mass M_α und charge Z_α,
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time independent potentials

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P.M.A Dirac, *Proceedings of the Royal Society* **A123**, 714 (1929)

separation of variables

$$i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right)\Psi(\vec{r},t)$$

time-independent potential

ansatz: $\Psi(\vec{r},t) = A(t)\psi(\vec{r})$

$$i\hbar\frac{\partial A(t)}{\partial t}\,\psi(\vec{r}) = A(t)\,E\,\psi(\vec{r}) = A(t)\,\left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right)\psi(\vec{r})$$

$$A(t) = A_0\,e^{-iEt/\hbar}$$

$$\left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right)\psi(\vec{r}) = E\,\psi(\vec{r})$$

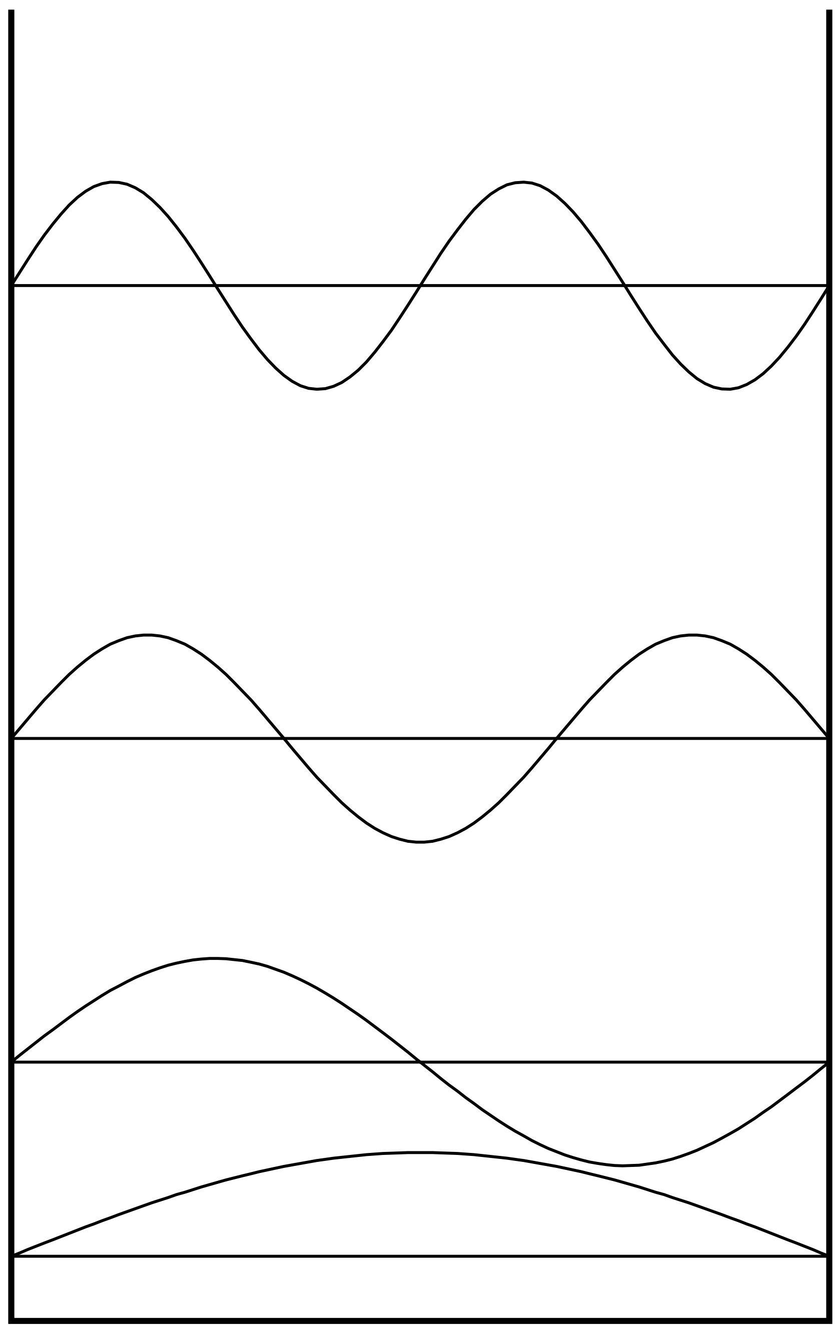
time-independent Schrödinger equation

(eigenvalue problem)

general solution: linear combination of eigenstates

$$\Psi(\vec{r},t) = \sum_n a_n\,e^{-iE_nt/\hbar}\,\psi_n(\vec{r})$$

particle in a box



boundary conditions ⇒ **quantization**

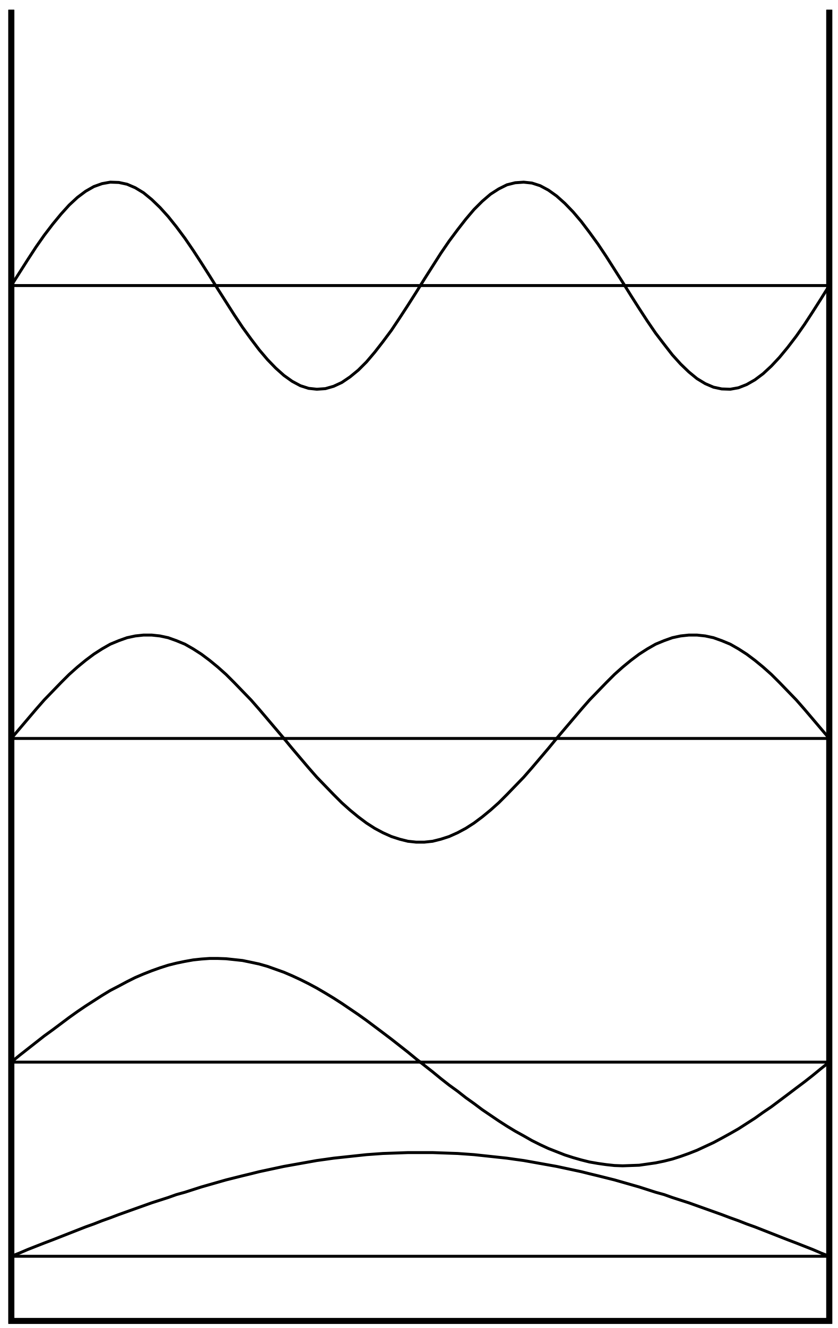
$$E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2$$

$$\varphi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi\, x}{L}\right)$$

discrete energies
zero-point energy
increasing number of nodes

symmetry of potential
symmetry of solutions (density)
even/odd eigenfunctions

particle in a box



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Nobel Prize in Chemistry 2023