Exercise Sheet 12 due 30 January

1. *time-dependent perturbation*

Consider an infinitely deep potential well of width L_z centered around $z = 0$ with an electron that is initially in the ground state. From time $t = 0$ to $t = T$ we apply a uniform electric field across the well, adding a term αz to the Hamiltonian, which is thus time dependent. We assume that the field is quite weak.

i. Using first-order time-dependent perturbation theory, derive an expression for the probability of finding the electron in the first excited state of the well after the field is switched off.

ii. What is the largest possible value for this probability? For what duration T is it attained?

iii. What is the probability that the electron will be found in the second excited state of the well after the field is switched off?

2. *Fermi's Golden Rule*

An electron is in the first excited state of a one-dimensional, infinitely deep potential well, with potential $V(z) = 0$ for $-L_z/2 < z < L_z/2$ and infinite otherwise. An oscillating electric field of the form

$$F(t) = F_0 \left(e^{-i\omega t} + e^{i\omega t}\right) = 2F_0 \cos(\omega t)$$

is applied along the z direction for a large but finite time, leading to a perturbing Hamiltonian during that time of the form

$$\hat{H}_p(t) = eF(t)z = \hat{H}_{po} \left(e^{-i\omega t} + e^{i\omega t}\right)$$

i. Consider the four lowest states of this well and presume that we are able to tune the frequency ω arbitrarily accurately to any frequency we wish. For each conceivable transition to another one of those states, say whether that transition is possible or essentially impossible, given an appropriate choice of frequency.

ii. What qualitative difference, if any, would it make if the well was of finite depth (though still considering only the first four states, all of which we presume to be bound in this finite well)?

Hint: Symmetry!