

Exercise Sheet 11 due 22 January1. *non-degenerate perturbation theory*

Consider an electron in a one-dimensional potential well of width L , with infinitely high barriers at $z=0$ and $z=L$. The potential energy inside the potential well is parabolic, of the form $V(z) = u(z-L/2)^2$, where u is a real constant. This potential is presumed to be small compared to the energy E_1 of the lowest state of a simple rectangular potential well of the same width L .

Find an approximate expression, valid in the limit of small u , for the energy difference between the lowest and first excited states of this well in terms of u , L , and fundamental constants.

2. *degenerate perturbation theory*

Consider a cubic quantum box for confining an electron. The cube has length L on all three sides, with edges along the x , y , and z directions, and the walls of the box are presumed to correspond to infinitely high potential barriers. We assume that $(x, y, z) = (0,0,0)$ is the point in the *center* of the box.

- i. Write down the normalized wavefunctions for the ground state and the first, threefold-degenerate, excited state for an electron in this box.
- ii. Now presume that there is a perturbation $\hat{H}_1 = eFx$ applied (e.g., from an electric field F in the x direction). How does the energy of the ground-state and of the three-fold degenerate excited state change as a result of this perturbation, according to first-order degenerate perturbation theory?
- iii. Now presume that a perturbation $\hat{H}_1 = \alpha x^2$ is applied instead. (Such a perturbation could result, e.g., from a uniform fixed background charge density in the box.) Using first-order degenerate perturbation theory, what are the new eigenstates and eigenenergies arising from the three originally degenerate states?

useful integrals:

$$\int_{-\pi/2}^{\pi/2} \theta^2 \cos^2 \theta \, d\theta = \pi^3/24 - \pi/4$$

$$\int_{-\pi/2}^{\pi/2} \theta^2 \sin^2 2\theta \, d\theta = \pi^3/24 - \pi/16$$