

Exercise Sheet 10 due 11 January1. *hydrogen atom*

Consider the states of hydrogen with principal quantum numbers $n=1, 2,$ and 3 .

- i. What values of l and m are allowed?
- ii. How many nodes do the radial functions $u_{n,l}(r)$ and $R_{n,l}(r) = u_{n,l}(r)/r$ have? How do they behave for $r \rightarrow 0$ and what is their value at $r=0$?

2. *magnetic moment*

From classical magnetostatics we know that the magnetic moment due to an electrical current density \vec{j}_e is given by

$$\vec{m} = \frac{1}{2} \int \vec{r} \times \vec{j}_e d^3r .$$

- i. Given the quantum-mechanical probability current density

$$\vec{j} = \frac{\hbar}{2im_e} \left(\overline{\psi(\vec{r})} \vec{\nabla} \psi(\vec{r}) - \psi(\vec{r}) \vec{\nabla} \overline{\psi(\vec{r})} \right) ,$$

calculate the corresponding magnetic moment. Compare to the expectation value of the angular momentum operator $\vec{L} = \vec{r} \times \frac{\hbar}{i} \vec{\nabla}$.

- ii. What is the z-component of the magnetic moment for the following orbitals of the hydrogen atom $|n, l, m\rangle$: $|1, 0, 0\rangle$, $|2, 1, -1\rangle$, $|2, 1, 0\rangle$, $|2, 1, 1\rangle$, and $|3, 2, 1\rangle$. Express your results using the Bohr-magneton

$$\mu_B = \frac{e\hbar}{2m_e} .$$