Exercise Sheet 10 due 17 January 2019

1. *hydrogen atom*

Consider the states of hydrogen with principal quantum numbers n=1, 2, and 3.

i. What values of l are allowed?

ii. Plot the radial wave-functions $u_{n,l}(r)$ and $R_{n,l}(r) = u_{n,l}(r)/r$ for these wave functions. How do $u_{n,l}(r)$ and $R_{n,l}(r)$ behave for $r \to 0$? What are the values at $r = 0$?

iii. What is the probability of finding the electron in an infinitesimally small sphere of radius dr around the nucleus?

2. *magnetic moment*

From classical magnetostatics we know that the magnetic moment due to an electrical current density $\vec{j}_e$ is given by

$$\vec{m} = \frac{1}{2} \int \vec{r} \times \vec{j}_e \, d^3r \, .$$

i. Given the quantum-mechanical probability current density

$$\vec{j} = \frac{\hbar}{2im_e} \left(\overline{\psi(\vec{r})} \, \vec{\nabla} \, \psi(\vec{r}) - \psi(\vec{r}) \, \vec{\nabla} \, \overline{\psi(\vec{r})} \right) ,$$

calculate the corresponding magnetic moment. Compare to the expectation value of the angular momentum operator $\vec{L}$.

ii. What is the z-component of the magnetic moment for the following orbitals of the hydrogen atom $|n, l, m\rangle$: $|1, 0, 0\rangle$, $|2, 0, 0\rangle$, $|2, 1, -1\rangle$, $|2, 1, 0\rangle$, $|2, 1, 1\rangle$, and $|5, 3, 2\rangle$. Express your results using the Bohr-magneton

$$\mu_B = \frac{e\hbar}{2m_e} \, .$$