

Exercise Sheet 9 due 14 December1. *spherical harmonics*

- i. Using the spherical harmonics for $l = 1$ derived in the lecture, calculate $|Y(\vartheta, \varphi)|^2$ for $Y(\vartheta, \varphi) = \frac{1}{\sqrt{3}}(Y_{1,-1}(\vartheta, \varphi) + Y_{1,0}(\vartheta, \varphi) + Y_{1,1}(\vartheta, \varphi))$.
- ii. Write the spherical harmonics for $l = 1$ in Cartesian coordinates, i.e., replace $\sin \vartheta \cos \varphi$ by x/r , $\sin \vartheta \sin \varphi$ by y/r , and $\cos \vartheta$ by z/r .
- iii. Write the operators \hat{L}_x , \hat{L}_y , and \hat{L}_z in the space of the spherical harmonics with $l=1$, i.e., the 3×3 matrices $\langle l=1, m | \hat{L}_j | l=1, m' \rangle$. Do the same for \hat{L}_\pm and \hat{L}^2 .
- iv. Using the ladder operators starting from

$$Y_{2,-2}(\vartheta, \varphi) = \sqrt{\frac{15}{32\pi}} (\sin \vartheta)^2 e^{-2i\varphi}$$

derive the spherical harmonics for $l=2$ with $m > -2$:

$$Y_{2,-1}(\vartheta, \varphi) = \sqrt{\frac{15}{8\pi}} \sin \vartheta \cos \vartheta e^{-i\varphi}$$

$$Y_{2,0}(\vartheta, \varphi) = \sqrt{\frac{5}{16\pi}} (3(\cos \vartheta)^2 - 1)$$

$$Y_{2,1}(\vartheta, \varphi) = -\sqrt{\frac{15}{8\pi}} \sin \vartheta \cos \vartheta e^{i\varphi}$$

$$Y_{2,2}(\vartheta, \varphi) = \sqrt{\frac{15}{32\pi}} (\sin \vartheta)^2 e^{2i\varphi}$$

- v. Write the spherical harmonics for $l=2$ in Cartesian coordinates.