Exercise Sheet 4 due 9 November

1. *Airy functions*

Let $w(z) = \alpha\mathrm{Ai}(z)+\beta\mathrm{Bi}(z)$ be a general solution of the Airy differential equation $w''(z) = z\,w(z)$.

i. Show that

$$\varphi(x) = w\left(\sqrt[3]{\frac{2me\mathcal{E}}{\hbar^2}}\left(x-\frac{E}{e\mathcal{E}}\right)\right)$$

solves the time in-dependent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2} + e\mathcal{E}x\,\varphi(x) = E\varphi(x).$$

ii. Express the normalization integral $\int_{x_1}^{x_2}|\varphi(x)|^2\,dx$ in terms of the integral over the Airy function $\int_{z_1}^{z_2}|w(z)|^2\,dz =: c$, where $z_i = \sqrt[3]{2me\mathcal{E}/\hbar^2}(x_i-E/e\mathcal{E})$.

2. *triangular well*

Consider a triangular well with potential energy $V(x) = \begin{cases} \infty & \text{for } x \leq 0 \\ e\mathcal{E}\,x & \text{for } x > 0 \end{cases}$ and $\mathcal{E} > 0$.

Find the three lowest eigen-energies when the electric field strenght is $\mathcal{E} = 0.1$ V/Å.