Exercise Sheet 3 due 8 November 2018

1. *finite potential well*

Consider a potential well of width L and depth V_0. Write a program to find the eigenenergies of all bound states. Run your code for V_0=4 eV and L= 5, and 20 Å and plot the normalized eigenfunctions. Compare to the corresponding eigenenergies for an infinite potential well of width L.

2. *Potential barrier*

Consider a barrier, 5 Å thick and 2 eV high. An electron wave is incident on this barrier from the left (perpendicular to the barrier).

i. Plot the tunneling probability of the transmission of an electron from one side of this barrier to the other as a function of energy from 0 eV to 6 eV.

ii. Assuming an incoming wave e^{ikx} from the left, plot the modulus squared of the electron wave function from 1 Å to the left of the barrier to 1 Å to the right of the barrier at an energy corresponding to the first maximum in the transmission for energies above the barrier.

iii. Calculate the electron wave-length above the barrier for the energies where transmission is largest. Using this, give an explanation for the form of the transmission as a function of energy for energies above the top of the barrier.

3. *Advanced: Many steps*

Write a program that calculates the solution of the Schrödinger equation for a piece-wise constant potential

$$V(x) = \begin{cases} 0 & , \ x < x_0 \\ V_1 & , \ x_0 < x < x_1 \\ V_2 & , \ x_1 < x < x_2 \\ \vdots & , \\ V_N & , \ x_{N-1} < x < x_N \\ V_{N+1} & , \ x_N < x \end{cases} \tag{1}$$

for a plane-wave of kinetic energy E coming from the left, i.e., $\varphi(x) = e^{ikx}$ for $x < x_0$. The solution for $x_{n-1} < x < x_n$ is written as

$$\varphi(x) = A_n e^{ik_n x} + B_n e^{-ik_n x}\,, \tag{2}$$

where the amplitudes A_n and B_n are determined from the matching conditions. The program should read the energy E and the potential parameters x_i, V_i. It should plot the potential $V(x)$, calculate the amplitudes, and draw the wavefunction in the form $E + |\varphi(x)|^2$ into the plot of the potential (see the lecture slides for an example). Test your code for the problem above.