Exercise Sheet 2 due 24 October

1. *probability current density*

Consider a plane wave $\Psi(\vec{r}, t) = e^{i(\vec{k}\cdot\vec{r}-\omega(\vec{k})t)}$. What is the corresponding probability current density? Verify the continuity equation.

2. *time evolution for a particle in a box*

Consider an electron in an infinite potential well of width L. Suppose that the wave function of the electron at time t=0 is $\Psi(x, t = 0) = A\sin^3(\pi x/L)$.

i. Check that $\Psi(x, t = 0)$ fulfills the boundary conditions and determine A so that is normalized.

ii. Write down the wave function for arbitrary time t and show that it is normalized for any t. (Hint: $4\sin^3(x) = 3\sin(x) - \sin(3x)$)

iii. Calculate the probability density $|\Psi(x, t)|^2$ as a function of time and plot it. For what times Δt is $|\Psi(x, t + \Delta t)|^2 = |\Psi(x, t)|^2$? What frequency does that correspond to? Does it change for superpositions of more states?

3. *wave packets*

i. Assume an electron is described by a Gaussian wave packet of width $\sigma_x =$ 1 Å. After what time has the width doubled. Now consider a dust particle of mass 1 mg described by a wave packet of width 1 μm. How many years does it take for the width to increase by 1 nm?

ii. Plot the probability density for a Gaussian wave packet at different times, and observe its velocity and broadening.