**Exercise Sheet 1** due 19 October

Course web site: `http://www.cond-mat.de/teaching/QM/`
Hand in exercises on due day *before* the lecture

1. *separation of variables*

Consider a two-dimensional box of sides $L$ with time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}\right)\Phi(x,y)=E\Phi(x,y)$$

and boundary conditions $\Phi(x,y)=0$ for $x\in\{0,L\}$, and $y\in\{0,L\}$.

i. Use separation of variables to find the eigenstates of this box in terms of the eigenstates $\varphi_n(x)=\sqrt{2/L}\,\sin(n\pi x/L)$ of a one-dimensional particle in a box of width $L$.

ii. Write the allowed energy of the electron in this box in units of the ground-state energy, $E_1=\hbar^2\pi^2/(2mL^2)$, of a particle in a one-dimensional box of width $L$.

iii. Of all the states of the box, find the three that are lowest in energy. Different states of the same energy are called *degenerate*. Which of the states that you found are degenerate? Do you understand why those states have the same energy?

2. *linearity of Schrödinger equation*

Assume that $\Psi_1(r,t)$ and $\Psi_2(r,t)$ are solutions of the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(r,t)=\left(-\frac{\hbar^2}{2m}\Delta_r+V(r,t)\right)\Psi(r,t) \tag{1}$$

Show that any linear combination $c_1\Psi_1(r,t)+c_2\Psi_2(r,t)$, where $c_i$ are complex numbers, is also a solution of (1).

3. *particle in a box*

For an electron in an infinite well of width 0.8 nm calculate the energy of the ground state and the first excited state in SI units and in electron volts. What color is the light made of photons whose energy equals the difference of the eigenenergies of the electron? What width of the infinite well should you choose to obtain blue light?