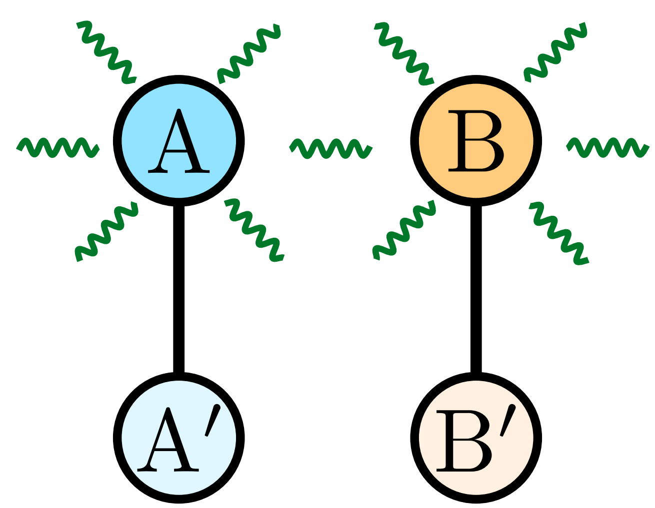
Markus Müller

Peter-Grünberg-Institut 2, Forschungszentrum Jülich
Institute for Quantum Information, RWTH Aachen
www.rwth-aachen.de/mueller-group

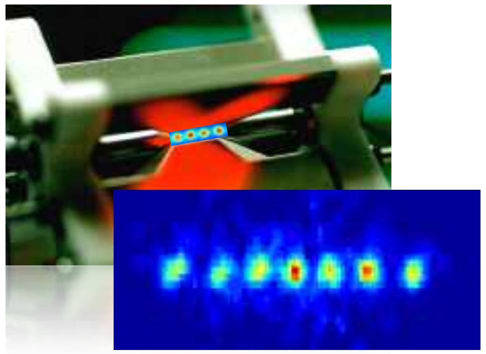
Quantifying Spatial Correlations in General Quantum Dynamics

Autumn School on Correlated Electrons:
Topology, Entanglement, and Strong Correlations

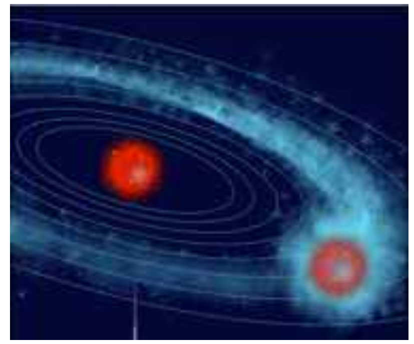


My background and research areas

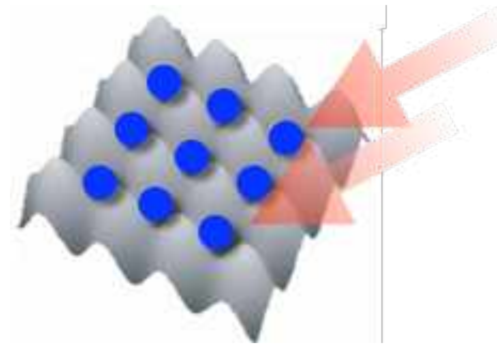
Trapped ions



Rydberg atoms

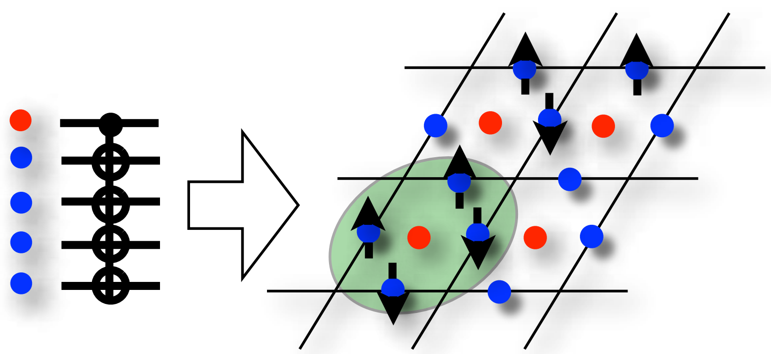


Neutral atoms

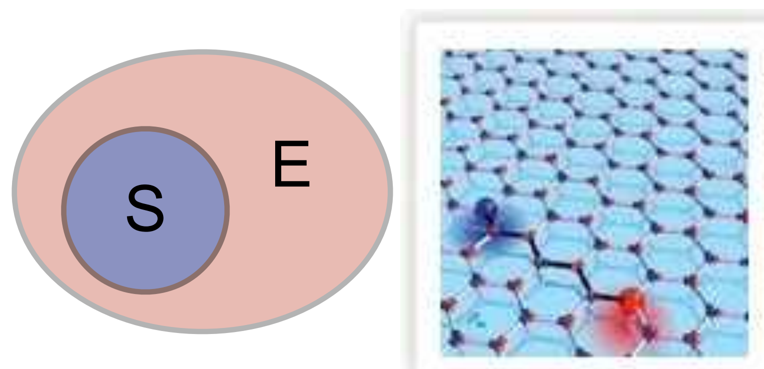


Trapped Rydberg ions

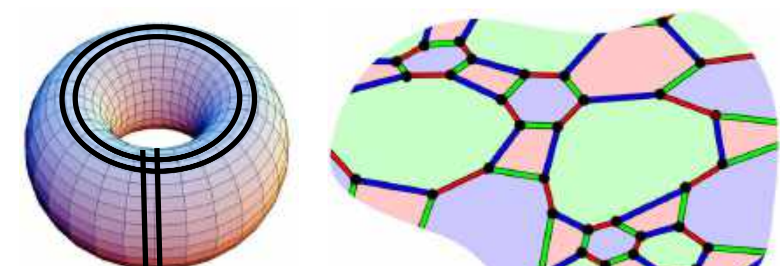
Research Lines & Applications



Quantum Information Processing
Quantum Gate Design
AMO System Engineering
Microscopic modelling



Quantum Simulations
Quantum Many-Body Systems
Reservoir Engineering
Topological Quantum Matter



QEC and Fault-Tolerant QC
Quantum Error Correction Protocols
Topological Quantum Codes
Decoders

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IOP Institute of Physics

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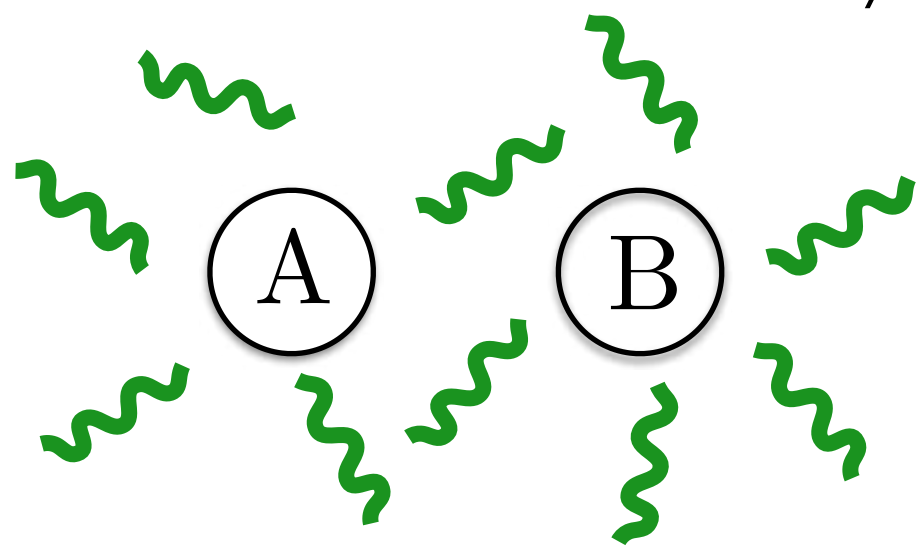
FAST TRACK COMMUNICATION

Quantifying spatial correlations of general quantum dynamics

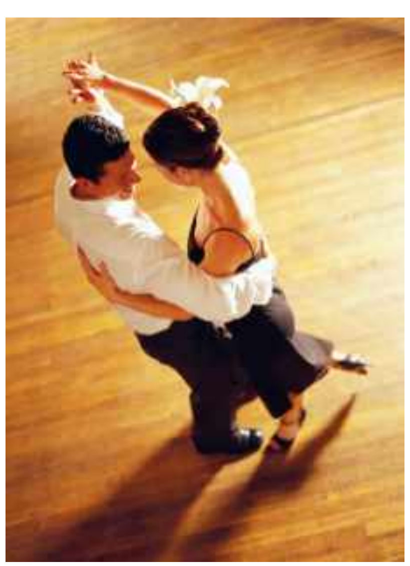
Ángel Rivas and Markus Müller
Departamento de Física Teórica I, Universidad Complutense, E-28040 Madrid, Spain

Dynamics of Quantum Systems

Correlated vs Uncorrelated Dynamics



Evolution of a part of a multipartite quantum system may depend on the evolution of the others



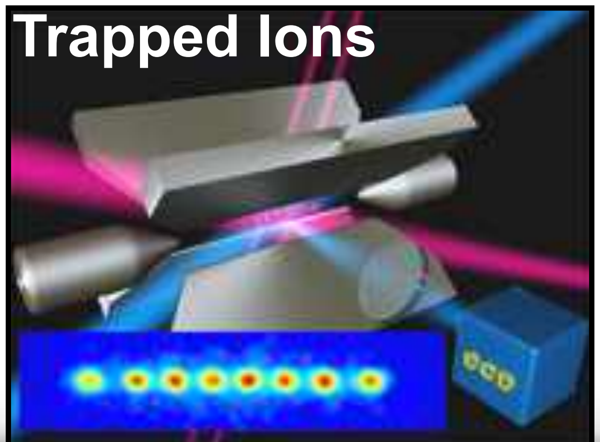
“Spatial” Correlations

Correlated Quantum Dynamics

Examples:

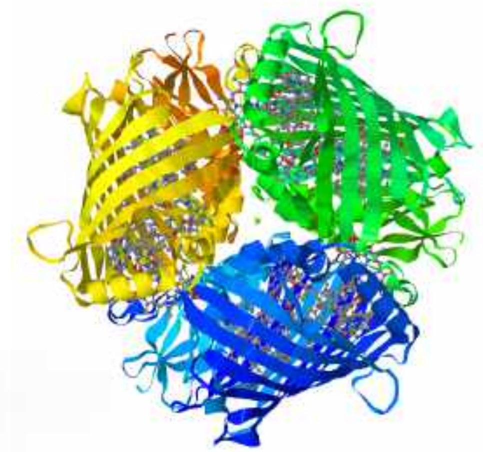
Any multipartite quantum system with interaction among their parties, or with a common bath

Quantum Computers and Simulators

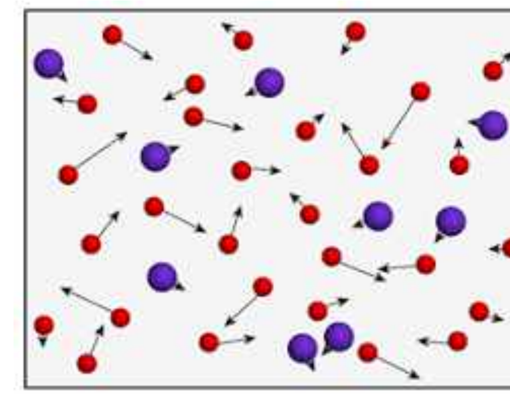


Blatt group (Innsbruck, Austria)

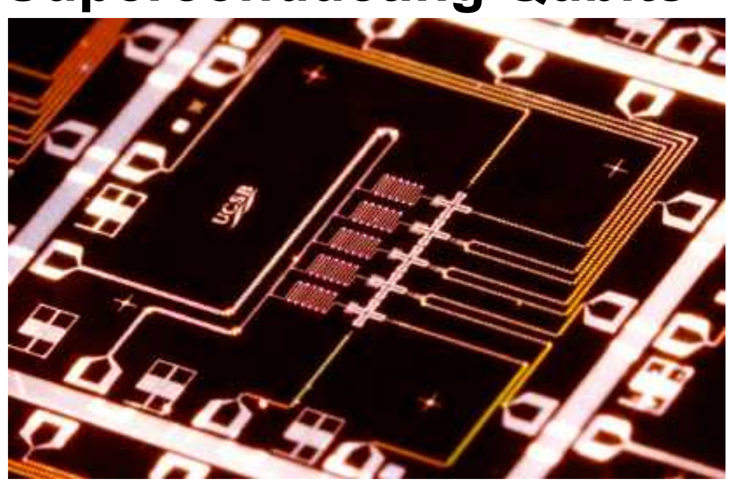
Biomolecules



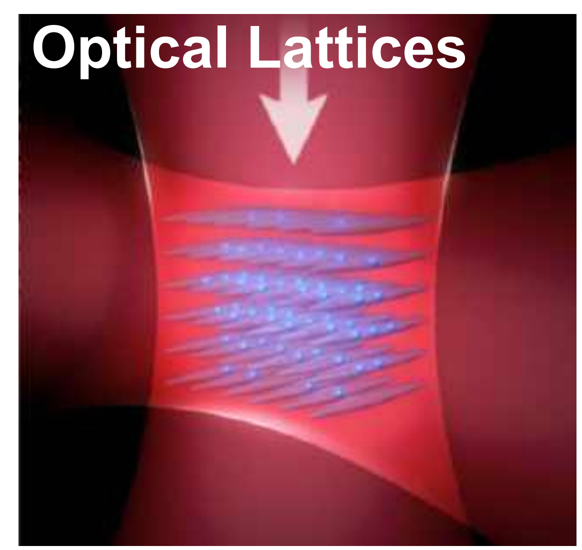
Atomic gases



Superconducting Qubits



Martinis group (UCSB, California)



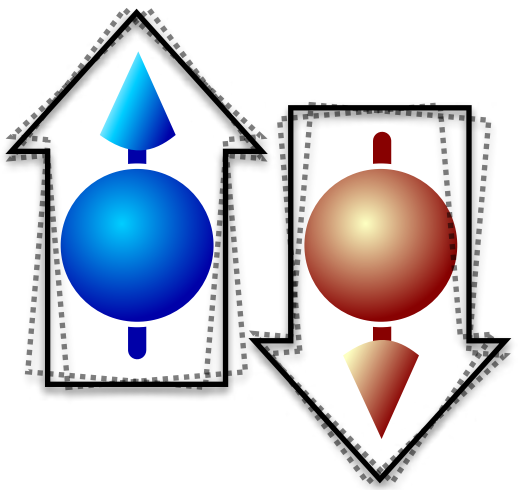
Ye group (NIST, Colorado)

Fundamental processes and applications,
e.g. performance of quantum information processing protocols

Harnessing dynamical correlations

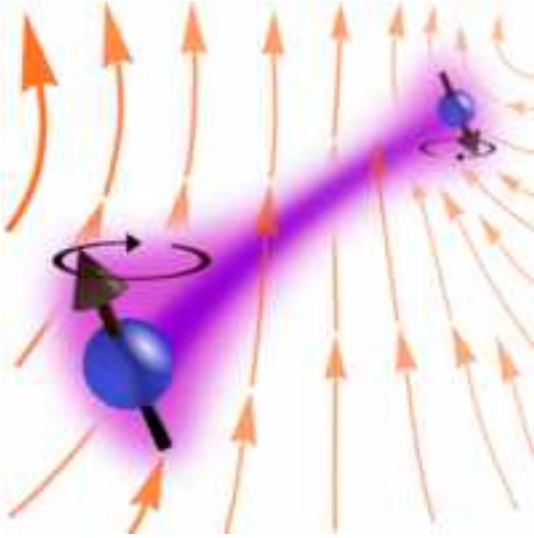
Protection of quantum states

Decoherence-Free Subspaces (DFS),
Dynamical decoupling, quantum control

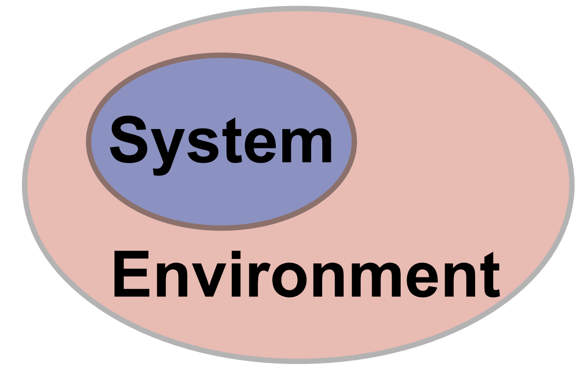


Quantum technology

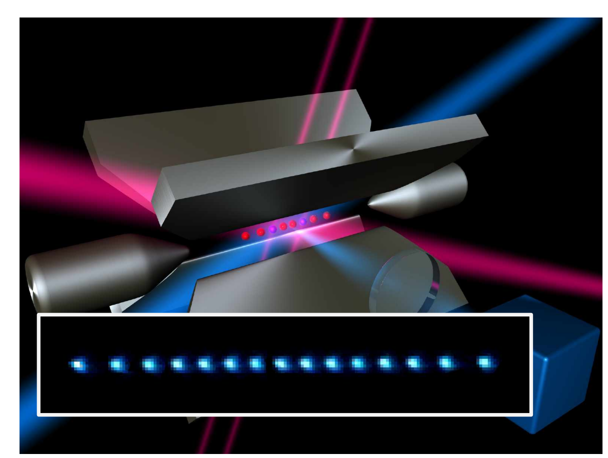
Entanglement-based magnetometry,
Quantum sensing



Outline of this lecture

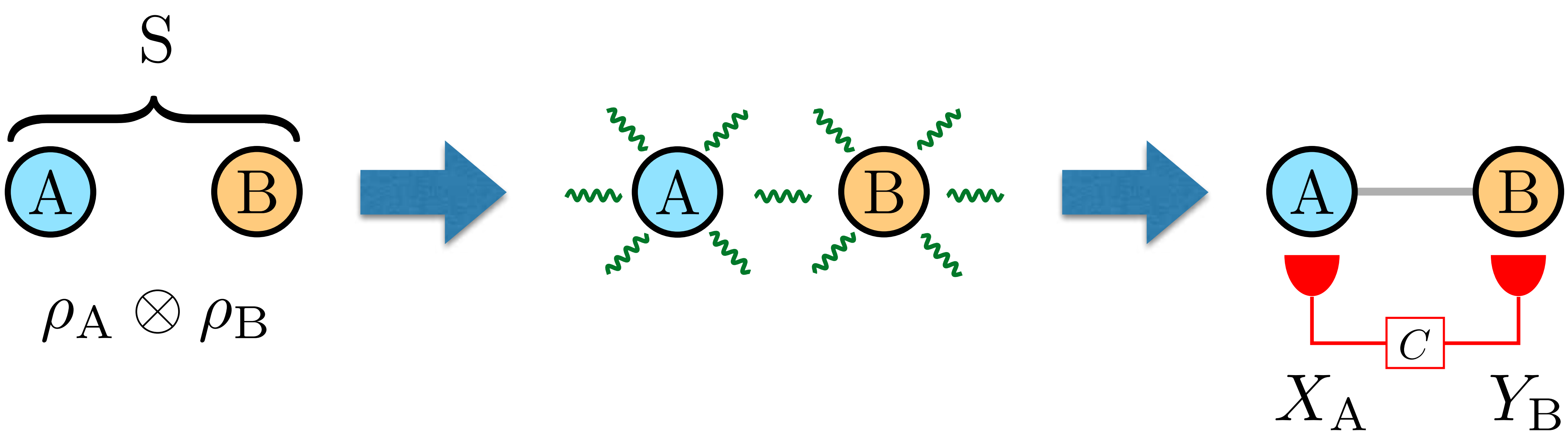


1) Quantifying correlations in quantum states
2) Dynamics in closed and open quantum systems
3) Quantifying spatial correlations in quantum dynamics
4) Physical application: radiating atoms
5) Experimental noise characterisation in a quantum computer
6) Harnessing spatially correlated noise



Quantifying Correlations in Dynamics

First attempt: correlations of observables



$$C(X_A, Y_B) = \langle X_A \otimes Y_B \rangle - \langle X_A \rangle \langle Y_B \rangle$$

If $C(X_A, Y_B) \neq 0$ → Correlated Dynamics

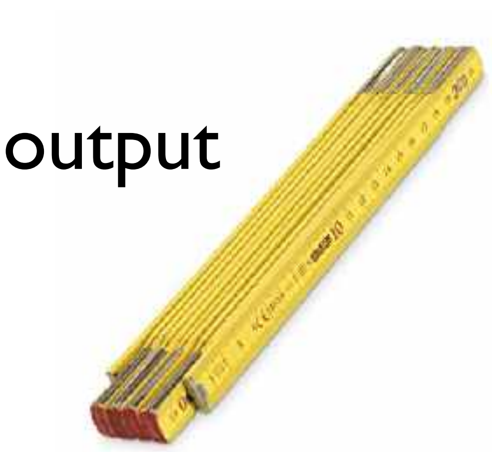
Not enough!

Quantifying Correlations in Dynamics

Introduce a quantifier to assess the amount of correlations in quantum dynamics in a general way, and independently of specific underlying situations or models

Wishlist

- No need for good guesses of test states
- No need for good guesses of test observables
- Quantitative measure - not a simple yes / no / I don't know output



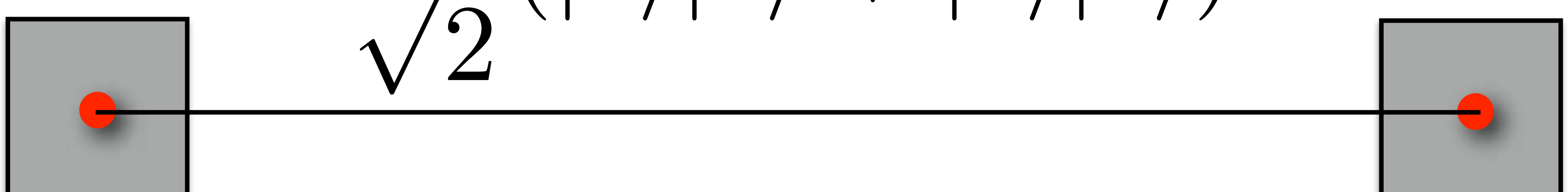
Correlations in quantum ~~dynamics~~ states

ALICE

BOB

Example:
Bell states (EPR pairs)

$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$$



Two-qubit entangled state

$$\frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right) \qquad \mathcal{O}_\mathrm{A} = Z_\mathrm{A} \text{ and } \mathcal{O}_\mathrm{B} = Z_\mathrm{B}$$

measurement
(in Z-basis)

$|0\rangle$ and $|1\rangle$

completely
random
measurement
outcomes
(+1 or -1)

50% - 50%

$$\rho_\mathrm{S}|_\mathrm{A} = \mathrm{Tr}_\mathrm{B}(\rho_\mathrm{S}) = \frac{1}{2}\Big(|0\rangle\langle 0|_\mathrm{A} + |1\rangle\langle 1|_\mathrm{A} \Big) \qquad \langle Z_\mathrm{A}\rangle = \mathrm{Tr}_\mathrm{A}(Z_\mathrm{A}\rho_\mathrm{S}|_\mathrm{A}) = 0$$

Two-qubit entangled state

$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) \qquad \mathcal{O}_{\mathrm{A}} = Z_{\mathrm{A}} \text{ and } \mathcal{O}_{\mathrm{B}} = Z_{\mathrm{B}}$$

measurement
(in Z-basis)

$|0\rangle$ and $|1\rangle$

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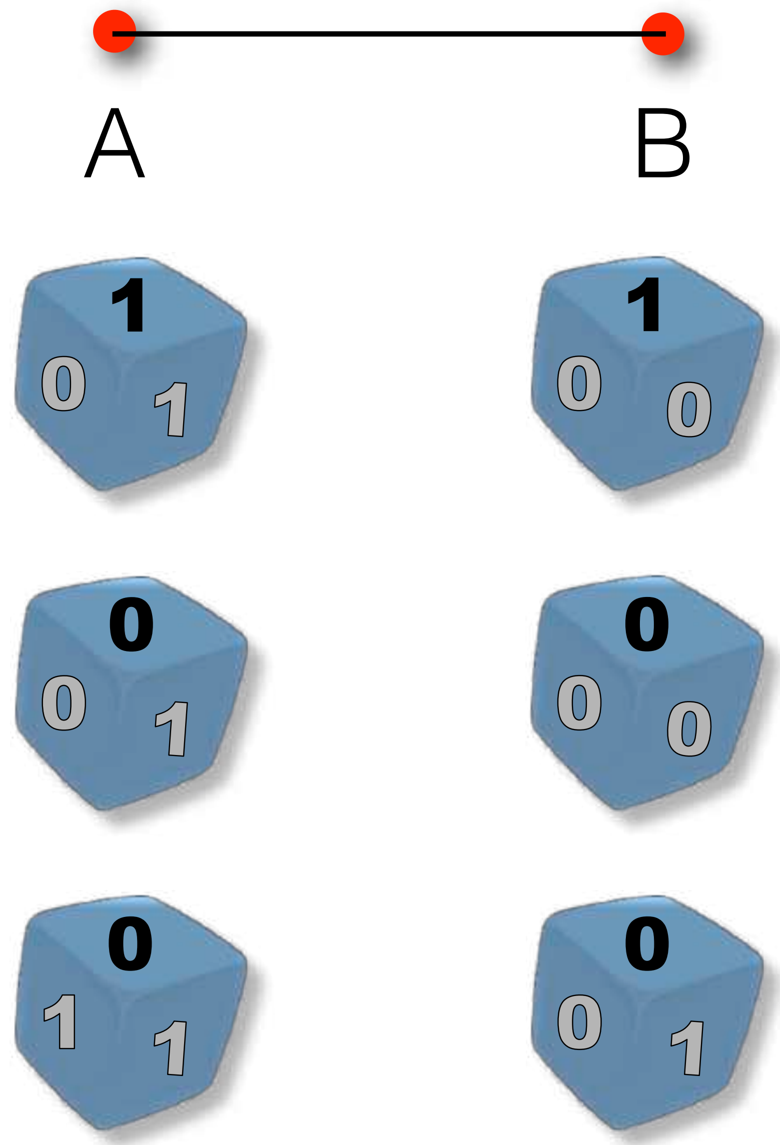
Two-qubit entangled state

$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$$



measurement (Z-basis)

measurement (Z-basis)

Completely **random**, but **perfectly correlated** +1/-1 outcomes

$$\langle Z_{\mathrm{A}} \otimes Z_{\mathrm{B}} \rangle = +1$$

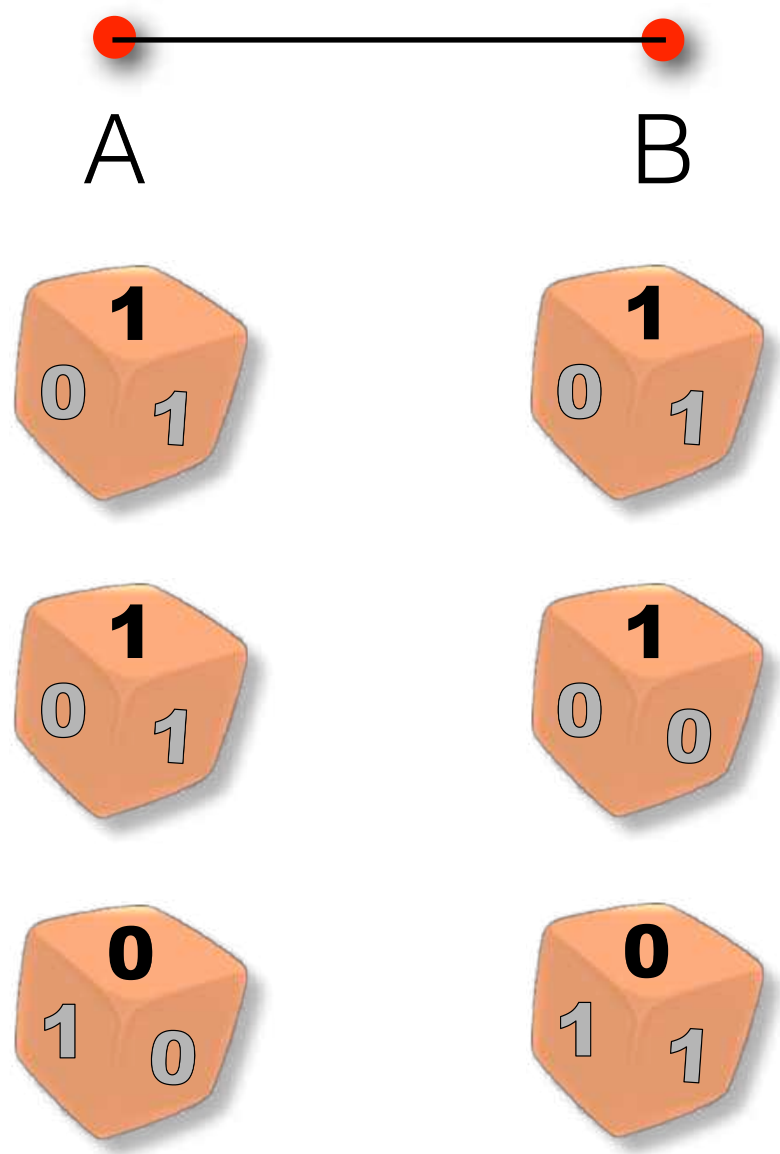
$$C(Z_A, Z_B) = \langle Z_A \otimes Z_B \rangle - \langle Z_A \rangle \langle Z_B \rangle = 1$$

Two-qubit entangled state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) = \frac{1}{\sqrt{2}}(|+\rangle|+\rangle + |-\rangle|-\rangle)$$

$$|\pm\rangle = \left(|0\rangle \pm |1\rangle\right)/\sqrt{2}$$

$$X|\pm\rangle = \pm|\pm\rangle$$



measurement (in X-basis)

measurement (in X-basis)

Completely **random**, but **perfectly correlated** +1/-1 outcomes

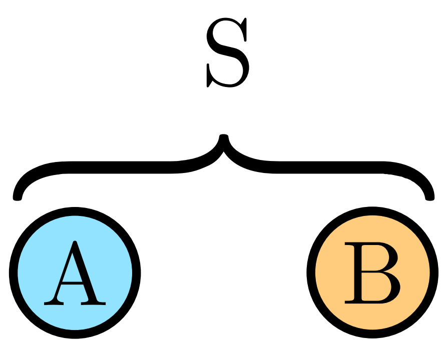
$$C(X_A, X_B) = \langle X_A \otimes X_B\rangle - \langle X_A\rangle\langle X_B\rangle = 1$$

Entangled: $|\Phi^+\rangle \neq |\psi_1\rangle_A \otimes |\psi_2\rangle_B$ with $|\psi_i\rangle = \alpha_i|0\rangle_i + \beta_i|1\rangle_i$

How correlated is a bi-partite state?

Quantum mutual information

generalises Shannon mutual information
(quantifying dependence between two random variables)



ρ_{S}

von Neumann entropy $S(\rho_{\mathrm{S}}) := -\mathrm{Tr}\rho_{\mathrm{S}} \log(\rho_{\mathrm{S}})$

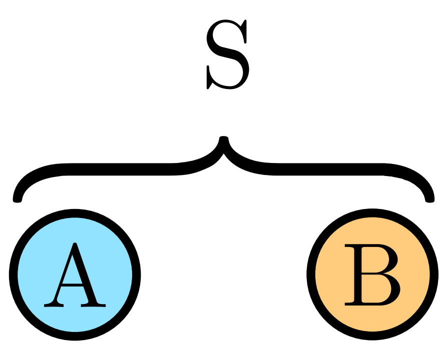
$$\rho_{\mathrm{S}} = \sum_i p_i \, |\psi_i\rangle\langle\psi_i|$$

with $p_i \geq 0$ and $\sum_i p_i = 1$

$$S(\rho_{\mathrm{S}}) = -\sum_i p_i \log p_i$$

How correlated is a bi-partite state?

Quantum mutual information



ρ_S

von Neumann entropy $S(\rho_\mathrm{S}) := -\mathrm{Tr}\rho_\mathrm{S}\log(\rho_\mathrm{S})$

Quantum mutual information

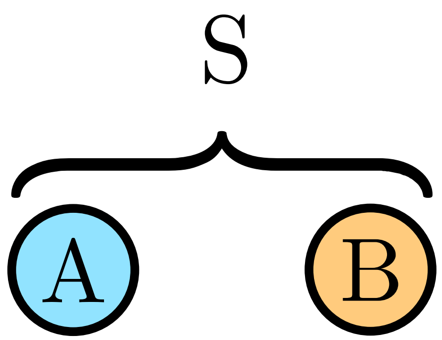
$$I(\rho_\mathrm{S}) = S(\rho_\mathrm{S}|_\mathrm{A}) + S(\rho_\mathrm{S}|_\mathrm{B}) - S(\rho_\mathrm{S})$$

Reduced density operators

$$\rho_\mathrm{S}|_\mathrm{A} = \mathrm{Tr}_\mathrm{B}(\rho_\mathrm{S}), \qquad \rho_\mathrm{S}|_\mathrm{B} = \mathrm{Tr}_\mathrm{A}(\rho_\mathrm{S})$$

How correlated is a bi-partite state?

Quantum mutual information



$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$$

ρ_{S}

von Neumann entropy $S(\rho_{\mathrm{S}}) := -\mathrm{Tr}\rho_{\mathrm{S}}\log(\rho_{\mathrm{S}})$

$$\rho_{\mathrm{S}}|_{\mathrm{A}} = \mathrm{Tr}_{\mathrm{B}}(\rho_{\mathrm{S}}) = \frac{1}{2}\Big(\,|0\rangle\langle 0|_{\mathrm{A}} + |1\rangle\langle 1|_{\mathrm{A}}\Big) \qquad S(\rho_{\mathrm{S}}|_{\mathrm{A}}) = S(\rho_{\mathrm{S}}|_{\mathrm{B}}) = \log 2$$

$$I(\rho_{\mathrm{S}}) = S(\rho_{\mathrm{S}}|_{\mathrm{A}}) + S(\rho_{\mathrm{S}}|_{\mathrm{B}}) - S(\rho_{\mathrm{S}}) = 2\log 2$$

Maximally entangled states

For any uncorrelated state $\rho_{\mathrm{S}} = \rho_{\mathrm{S}}|_{\mathrm{A}} \otimes \rho_{\mathrm{S}}|_{\mathrm{B}}$ we have $I(\rho_S) = 0$

Quantum Dynamics

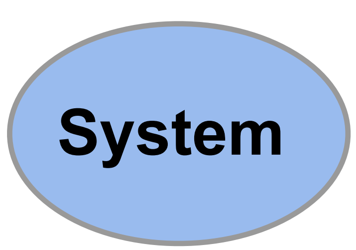
General time evaluation of a quantum system

Completely positive, trace-preserving (CPT) maps or Kraus maps

$$\mathcal{E}_{\mathrm{S}} : \rho_{\mathrm{S}} \mapsto \mathcal{E}_{\mathrm{S}}(\rho_{\mathrm{S}}) = \sum_i K_i \, \rho_{\mathrm{S}} \, K_i^\dagger$$

with a set of Kraus operators $\sum_i K_i^\dagger K_i = \mathbb{1}_{\mathrm{S}}$

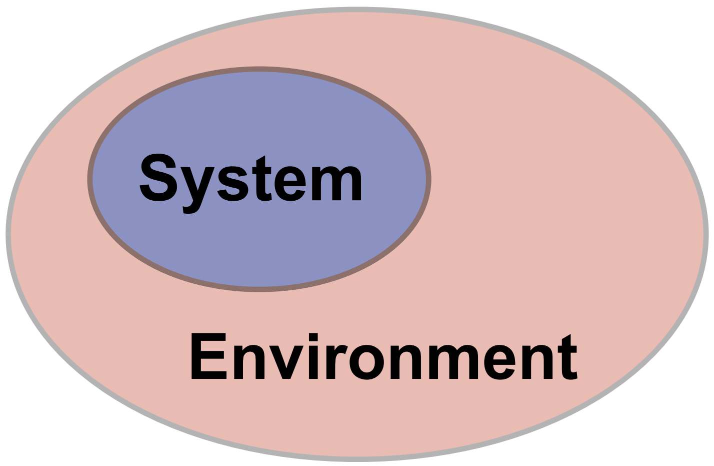
Closed system dynamics



$$\rho_{\mathrm{S}} \mapsto U_{\mathrm{S}} \, \rho_{\mathrm{S}} \, U_{\mathrm{S}}^\dagger$$

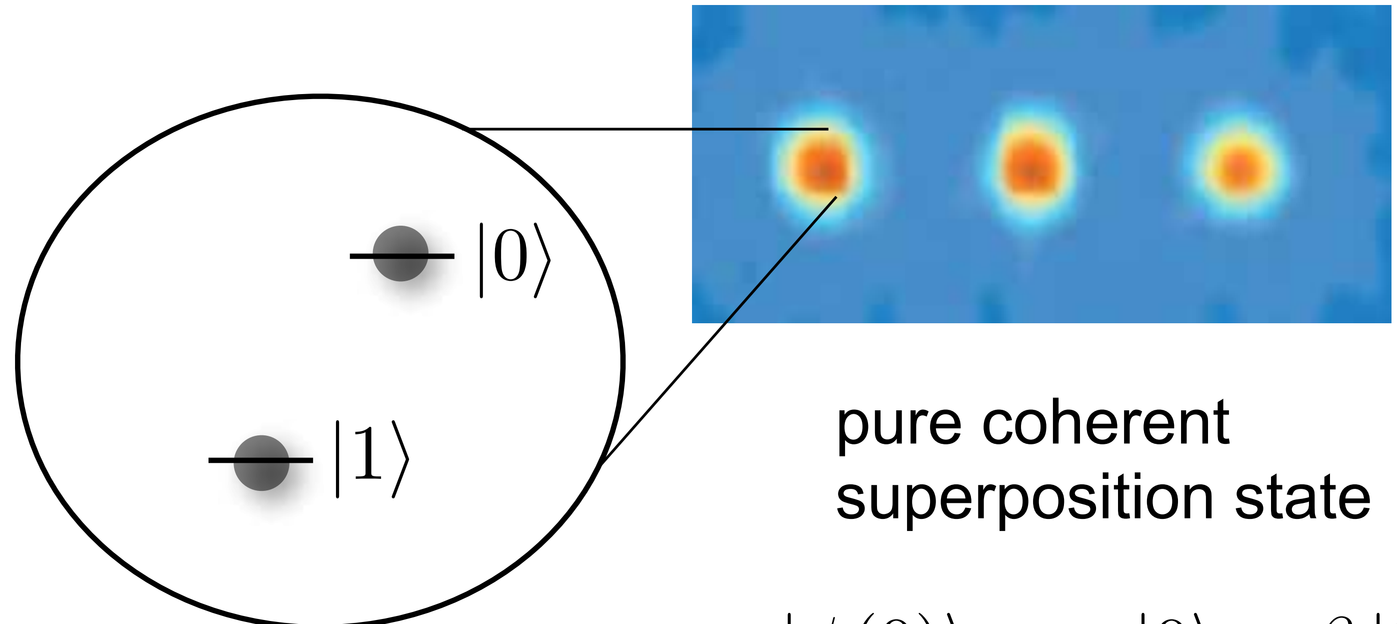
Open-system dynamics: dephasing channel

- Coupling to the environment causes decoherence:



e.g.
(effective) magnetic field fluctuations

$$H_G(t) = \frac{1}{2}\, B(t)\, Z.$$



pure coherent superposition state

$$|\psi(0)\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

dephasing dynamics

Superposition picks up unknown, fluctuating relative phases

$$\exp\left[\pm\mathrm{i}\int_0^t B(t')dt'\right]$$

$$\rho(t) = \int |\psi(t)\rangle\langle\psi(t)|\, P(B)\, dB =$$

$$= |\alpha|^2\,|0\rangle\langle 0| + |\beta|^2\,|1\rangle\langle 1| + e^{-\frac{1}{2}\gamma t}\left(\alpha\beta^*\,|0\rangle\langle 1| + \alpha^*\beta\,|1\rangle\langle 0|\right)$$

for Gaussian, stationary and delta-correlated noise

dephasing rate $\gamma = \left\langle [B(0)]^2 \right\rangle$

Open-system dynamics: dephasing channel

$$\rho(t) = \int |\psi(t)\rangle\langle\psi(t)|\, P(B)\, dB =$$

$$= \; |\alpha|^2 \, |0\rangle\langle 0| + |\beta|^2 \, |1\rangle\langle 1| + e^{-\frac{1}{2}\gamma t}\big(\alpha\beta^* \, |0\rangle\langle 1| + \alpha^*\beta \, |1\rangle\langle 0|\big)$$

From the evaluation for an arbitrary single-qubit state ρ_{S}

$$\mathcal{E}_{\mathrm{S}} : \rho_{\mathrm{S}} \mapsto (1{-}p)\rho_{\mathrm{S}} + pZ\rho_{\mathrm{S}}Z$$

we identify the Kraus map

$$\mathcal{E}_{\mathrm{S}} : \rho_{\mathrm{S}} \mapsto \mathcal{E}_{\mathrm{S}}(\rho_{\mathrm{S}}) = \sum_i K_i\, \rho_{\mathrm{S}}\, K_i^\dagger \qquad \textstyle\sum_i K_i^\dagger K_i = \mathbb{1}_{\mathrm{S}}$$

Kraus operators $K_0 = \sqrt{1{-}p}\,\mathbb{1}$ and $K_1 = \sqrt{p}Z$

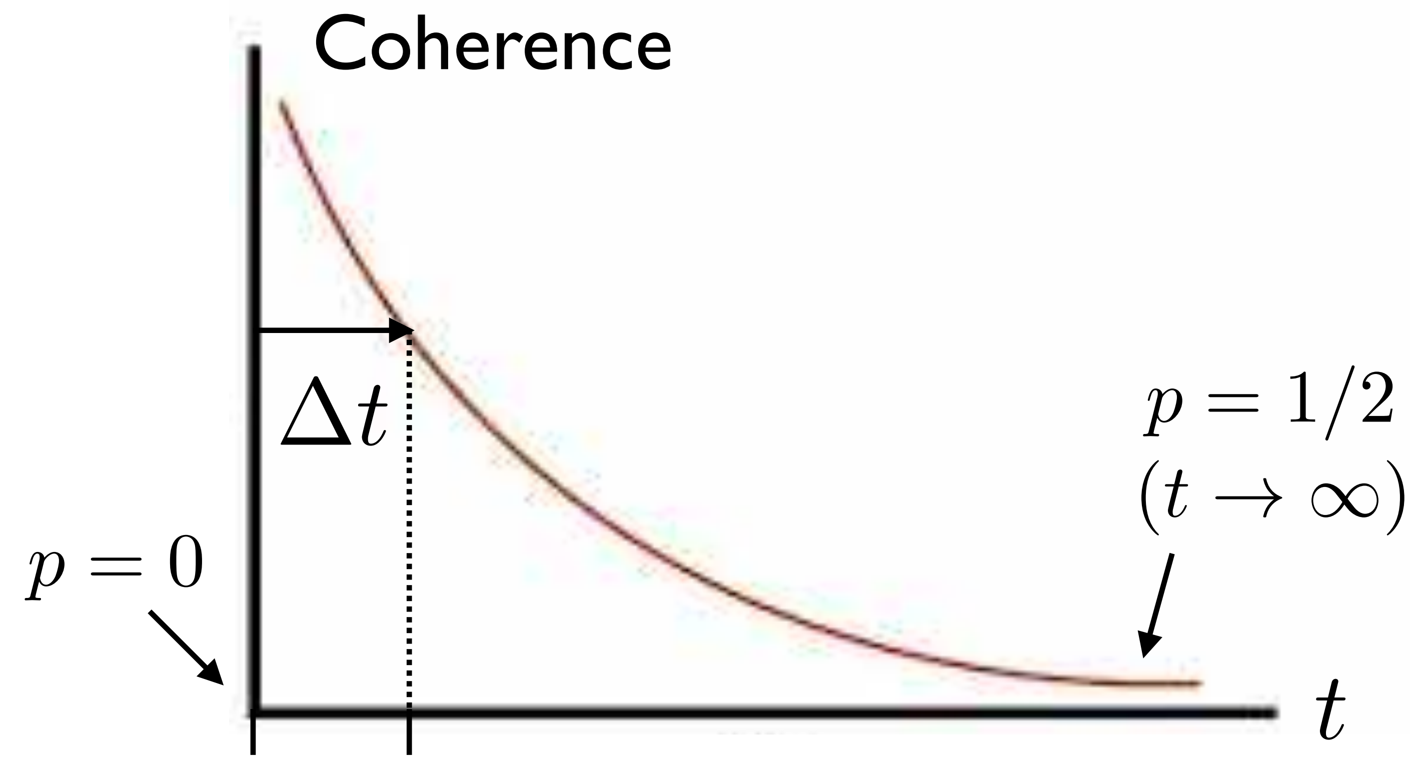
with $p = \frac{1}{2}(1 - e^{-\frac{1}{2}\gamma t})$

Open-system dynamics: dephasing channel

$\mathcal{E}_\mathrm{S} : \rho_\mathrm{S} \mapsto (1-p)\rho_\mathrm{S} + pZ\rho_\mathrm{S}Z$

Kraus operators $K_0 = \sqrt{1-p}\,\mathbb{1}$ and $K_1 = \sqrt{p}Z$

with $p = \frac{1}{2}(1 - e^{-\frac{1}{2}\gamma t})$



For a small time step

$$p \simeq \frac{1}{4}\gamma(\Delta t)$$

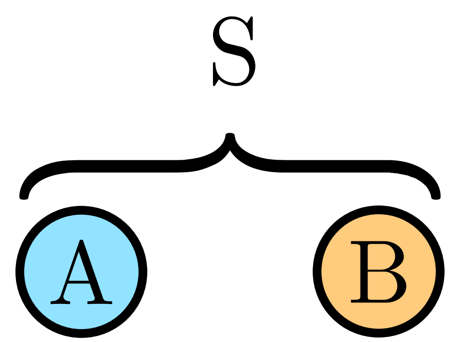
one recovers the quantum master equation in Lindblad form

$$\dot{\rho} \simeq \frac{\rho(t+\Delta t) - \rho(t)}{\Delta t} = \frac{\gamma}{4}(Z\rho Z^\dagger - \frac{1}{2}\{\rho, Z^\dagger Z\})$$

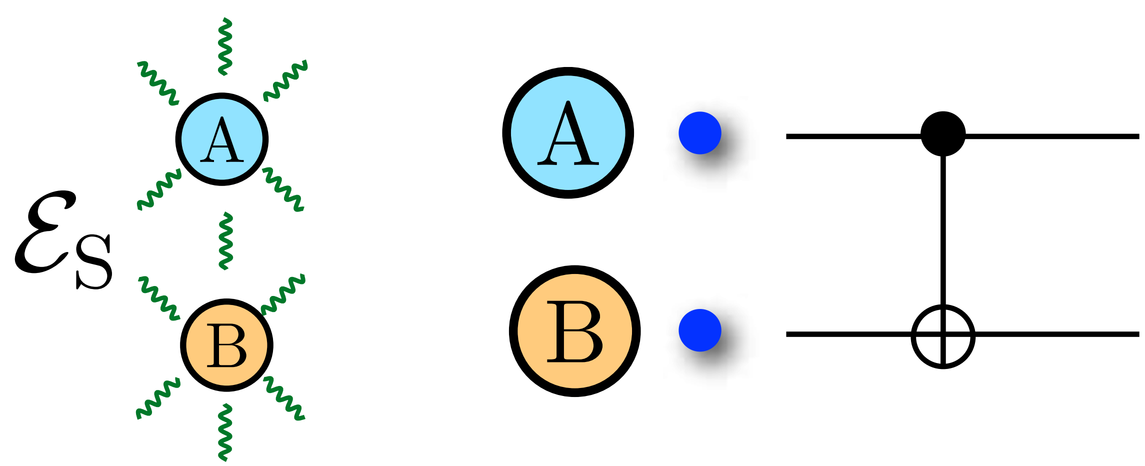
Correlated quantum dynamics

Focus on bi-partite systems

$$\dim(\mathcal{H}_\mathrm{A}) = \dim(\mathcal{H}_\mathrm{B}) = d$$

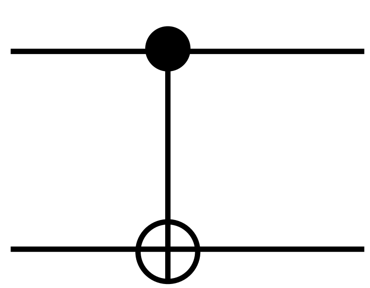


Example: 2-qubit entangling gate $\mathrm{CNOT} = |0\rangle\langle 0|_\mathrm{A} \otimes \mathbb{1}_\mathrm{B} + |1\rangle\langle 1|_\mathrm{A} \otimes X_\mathrm{B}$



$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|0\rangle$$



$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$$

Outputs a (maximally) correlated state

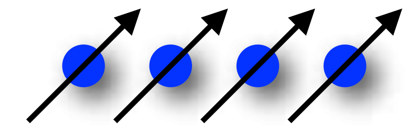
⇨ Correlated quantum dynamics

Correlated quantum dynamics

Spatially correlated, *global* dephasing on an N-qubit register $\mathcal{E}_{\mathrm{S}} \neq \otimes_k \mathcal{E}_k$

$$H_G(t) = \frac{1}{2} B(t) \sum_k Z_k$$

Test state $|\psi(0)\rangle = \otimes_k |+\rangle_k$

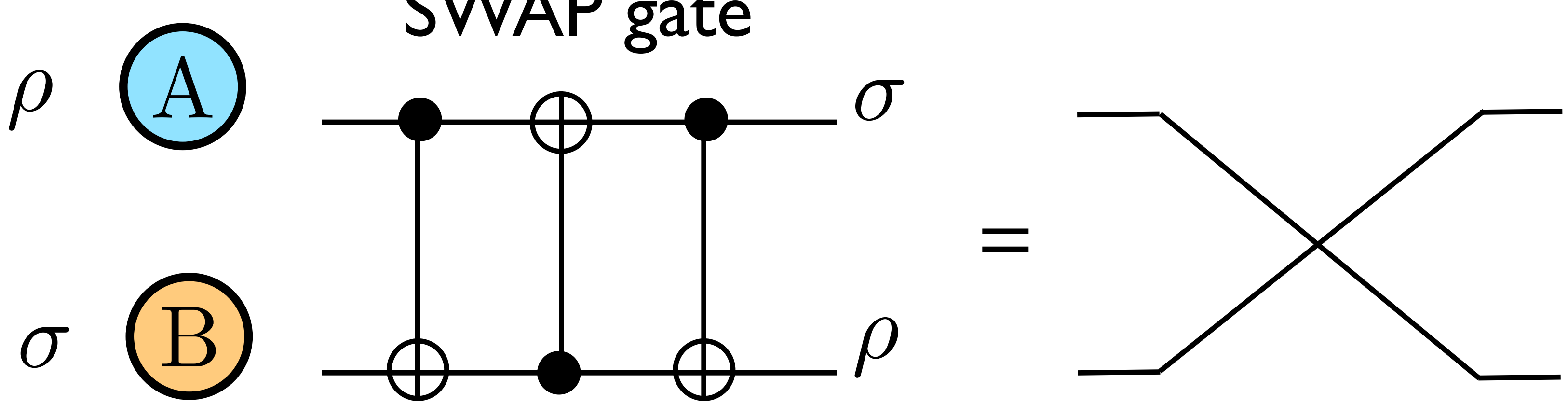


… will build up correlations
under the correlated dynamics $\mathcal{E}_{\mathrm{S}}$

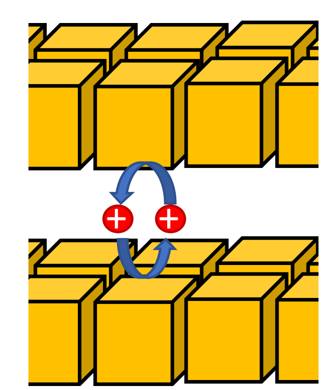
There are highly correlated quantum processes that do not create correlations

$$U_{\mathrm{SWAP}}(\rho \otimes \sigma) U_{\mathrm{SWAP}}^{\dagger} = \sigma \otimes \rho$$

SWAP gate



Real particle exchange, correlated hopping

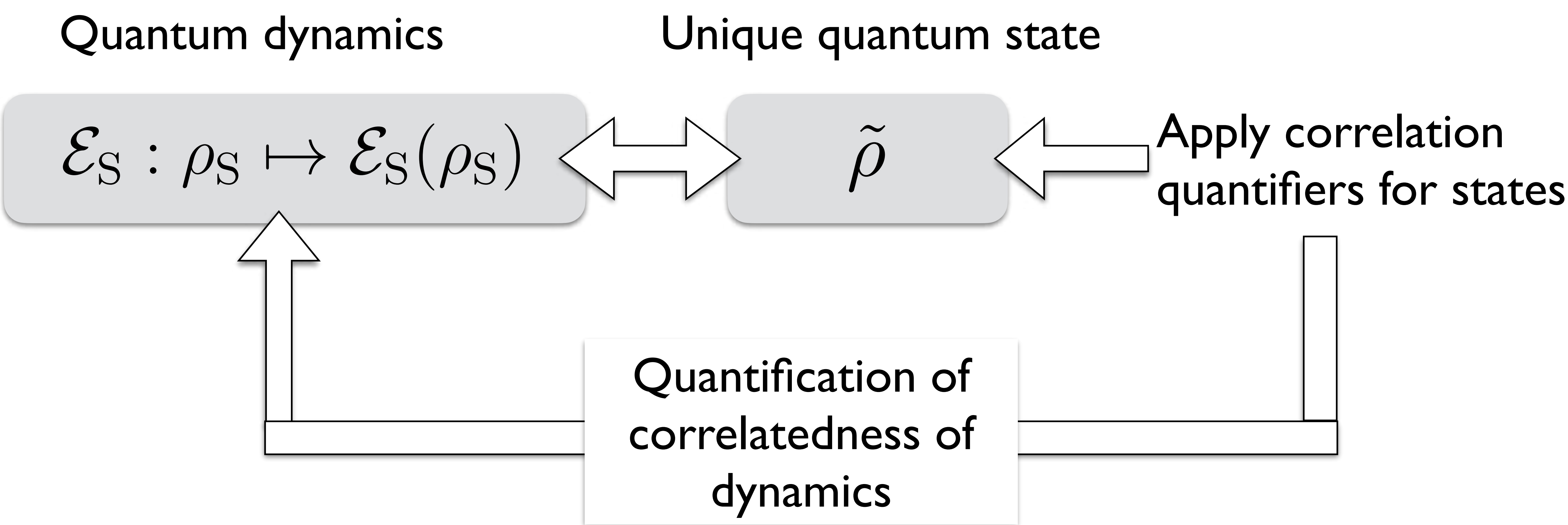


e.g melting & re-crystalisation of ion Coulomb crystals

How correlated are these?

Rigorous quantification of correlations in quantum dynamics

Basic idea



Choi-Jamiołkowski Isomorphism (1972)

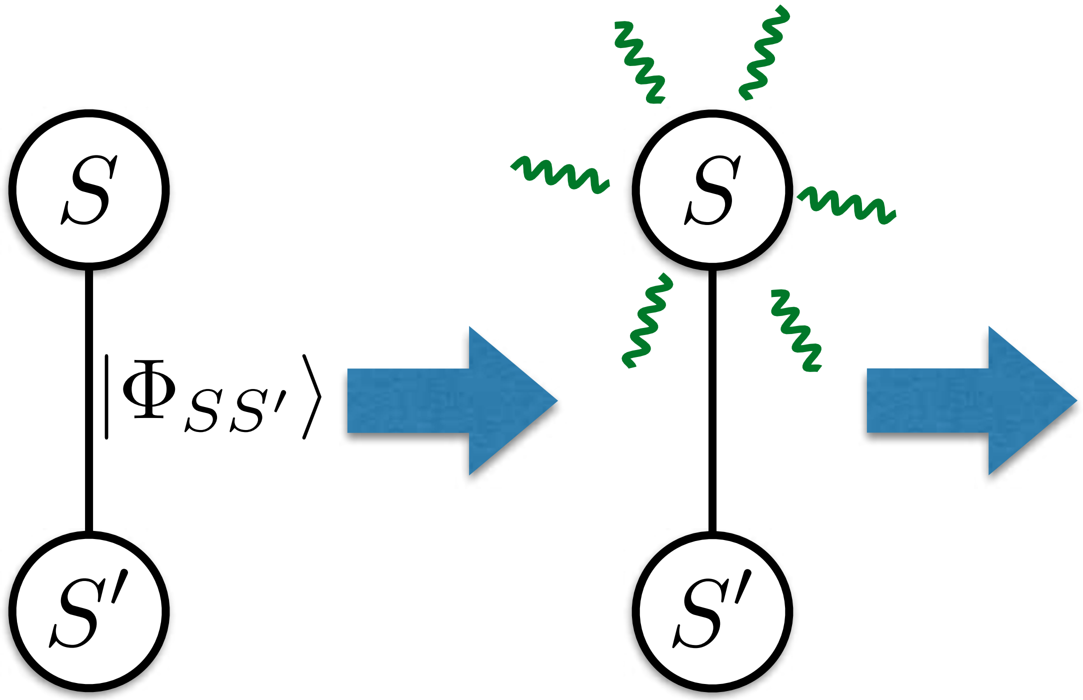
$$\mathcal{E}_{\mathrm{S}} : \rho_{\mathrm{S}} \mapsto \mathcal{E}_{\mathrm{S}}(\rho_{\mathrm{S}})$$

Physical system

$\dim(S) = d$

'copy' of system S

$\dim(S') = d$



Choi-Jamiołkowski state

$$\rho_{\mathrm{S}}^{\mathrm{CJ}} := \mathcal{E}_{\mathrm{S}} \otimes \mathbb{1}_{\mathrm{S'}}\left(|\Phi_{\mathrm{SS'}}\rangle\langle\Phi_{\mathrm{SS'}}| \right)$$

Maximally entangled state:

$$|\Phi_{SS'}\rangle = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} |jj\rangle_{SS'}$$

Man-Duen Choi

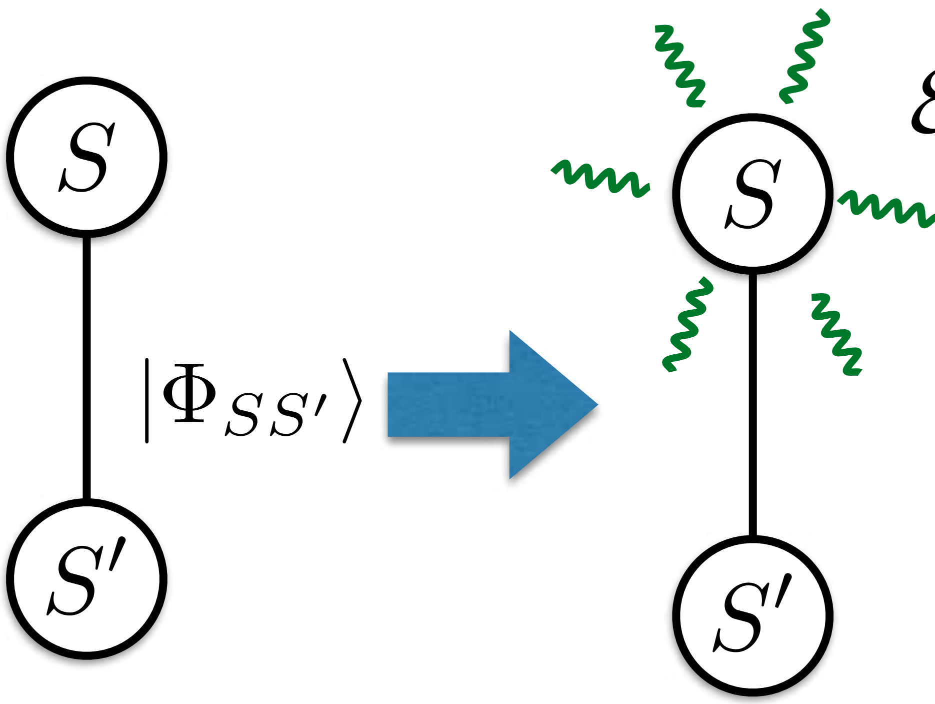
State-evolution correspondence (one-to-one)

$$\text{dynamics} \Leftrightarrow \rho_{\mathrm{CJ}}$$

Andrzej Jamiołkowski

Choi-Jamiołkowski Isomorphism

Example: Single-qubit dephasing channel $\dim(S) = 2$



$\mathcal{E}_\mathrm{S} : \rho_\mathrm{S} \mapsto (1-p)\rho_\mathrm{S} + pZ\rho_\mathrm{S}Z$

$$|\Phi_{SS'}\rangle = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} |jj\rangle_{SS'} = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$$

$$\rho_\mathrm{S}^\mathrm{CJ} := \mathcal{E}_\mathrm{S} \otimes \mathbb{1}_{\mathrm{S'}}\big(\, |\Phi_{\mathrm{SS'}}\rangle\langle\Phi_{\mathrm{SS'}}|\,\big)$$

$$= \frac{1}{2}\Big(\, |00\rangle\langle 00|_{\mathrm{SS'}} + |11\rangle\langle 11|_{\mathrm{SS'}}\,\Big) + \frac{1}{2}(1-2p)\Big(\, |00\rangle\langle 11|_{\mathrm{SS'}} + |11\rangle\langle 00|_{\mathrm{SS'}}\,\Big)$$

Construction of the correlation measure

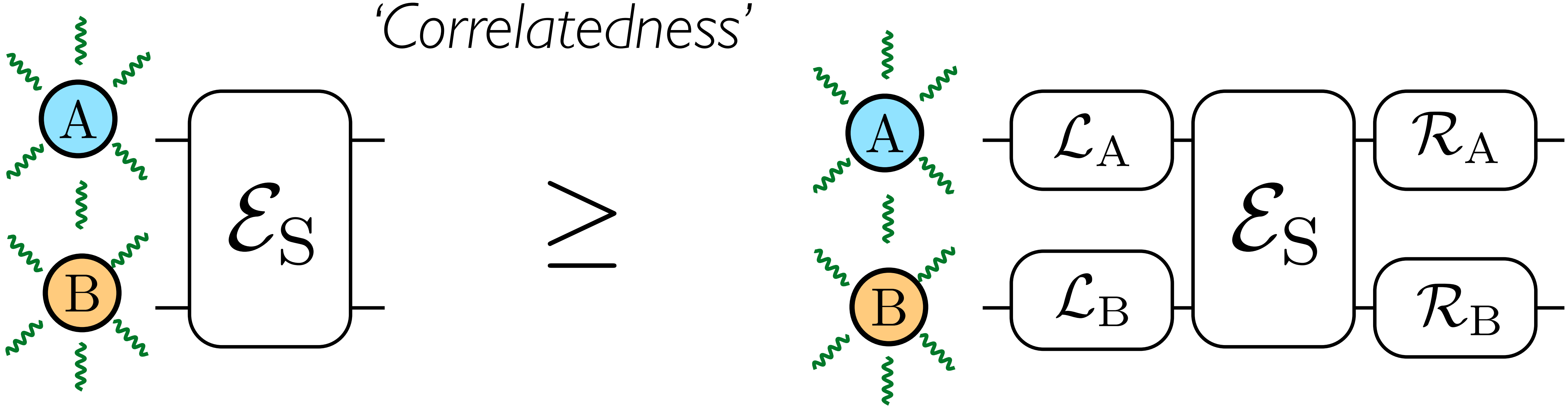
Resource-theory approach: Correlations as a *resource*

Correlated dynamics = resource to perform whatever task which can't be implemented solely by (composing) uncorrelated dynamics $\mathcal{E}_{\mathrm{A}} \otimes \mathcal{E}_{\mathrm{B}}$

Fundamental law of the resource theory

The amount of correlations of some dynamics does *not increase* under composition with uncorrelated dynamics $\mathcal{E}'_{\mathrm{S}} = (\mathcal{L}_{\mathrm{A}} \otimes \mathcal{L}_{\mathrm{B}})\mathcal{E}_{\mathrm{S}}(\mathcal{R}_{\mathrm{A}} \otimes \mathcal{R}_{\mathrm{B}})$

'Correlatedness'



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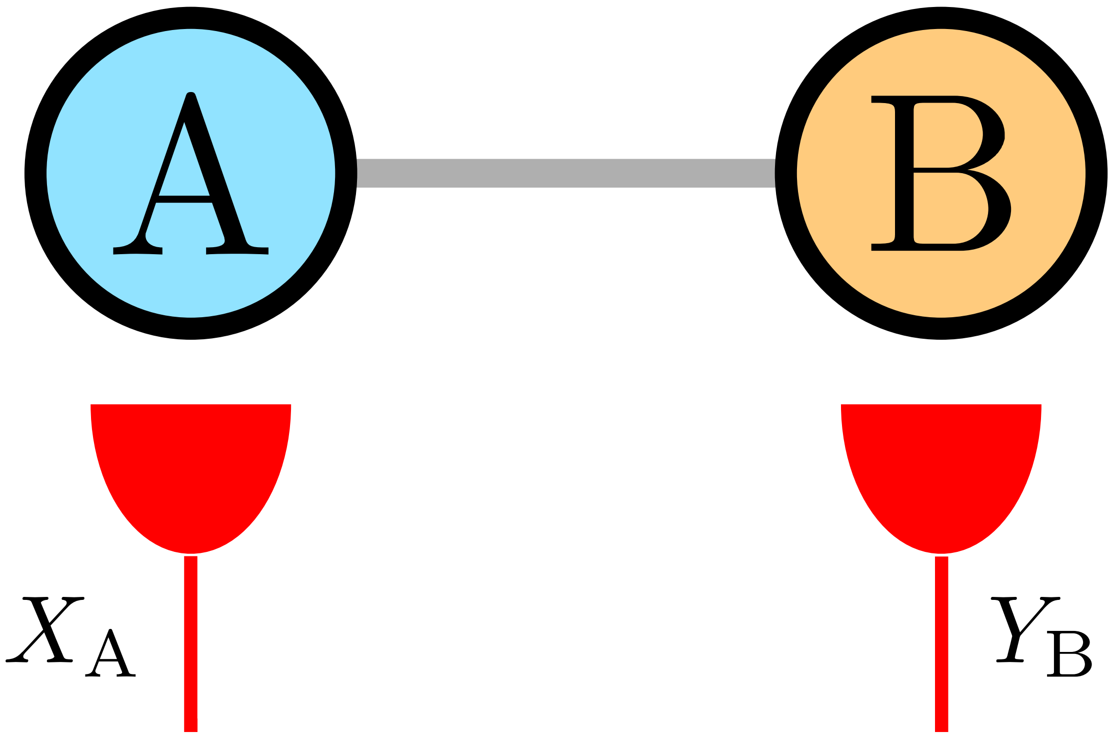
Good Quantifier: $\mathcal{I}(\mathcal{E}) \geq \mathcal{I}(\mathcal{E}')$ *partial order*

*L. Li, K. Bu and Z.-W. Liu, Quantifying the resource content of quantum channels: An operational approach, arXiv:1812.02572
*Y. Liu and X. Yuan, Operational Resource Theory of Quantum Channels, arXiv:1904.02680
*Z.-W. Liu and A. Winter, Resource theories of quantum channels and the universal role of resource erasure, arXiv:1904.04201.

Resource theory of entanglement

Entanglement as a resource …

does not increase under…



… **L**ocal **O**perations …

ALICE

BOB

… and **C**lassical **C**ommunication

(LOCC paradigm)

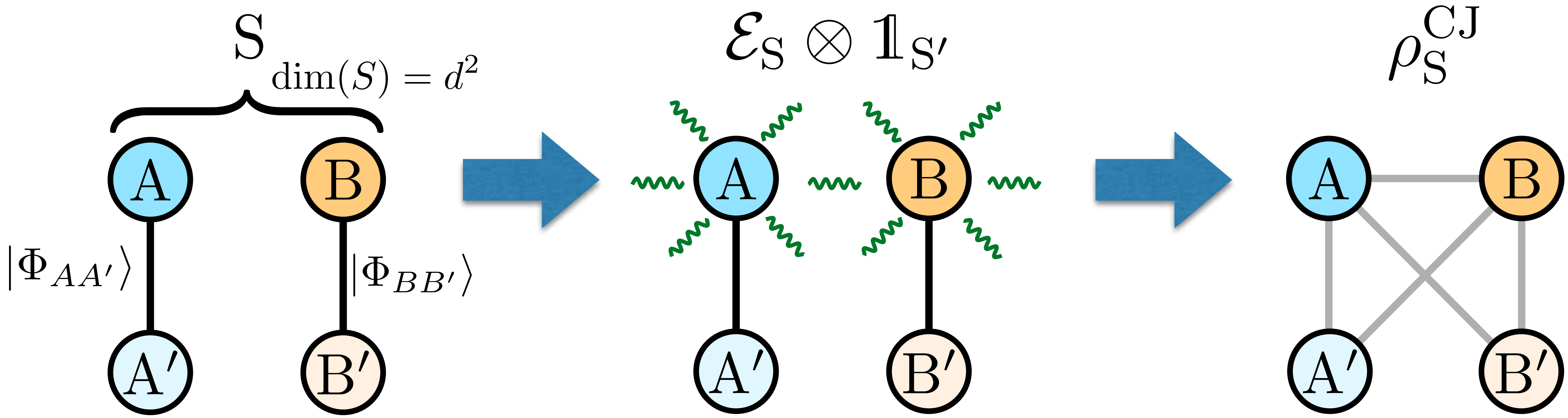
$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) \rightarrow\ U_{\mathrm{A}} \rightarrow \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + |0\rangle|1\rangle)$$

Correlation measure for dynamics

Bi-partite system:

$\dim(A) = \dim(B) = d$

Choi-Jamiołkowski Isomorphism



Maximally entangled state:

$$|\Phi_{\mathrm{SS'}}\rangle := \frac{1}{d}\sum_{j=1}^{d^2} |jj\rangle_{\mathrm{SS'}} = \frac{1}{d}\sum_{k,\ell=1}^{d} |k\ell\rangle_{\mathrm{AB}} \otimes |k\ell\rangle_{\mathrm{A'B'}}$$

Choi-Jamiołkowski state

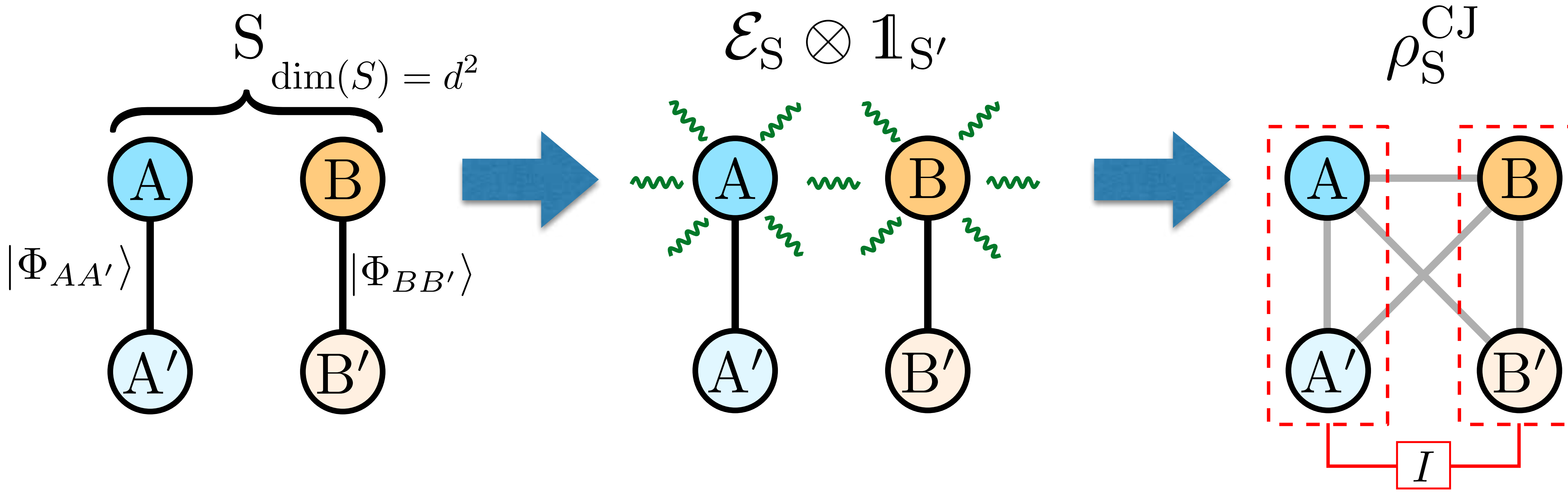
$$\rho_{\mathrm{S}}^{\mathrm{CJ}} := \mathcal{E}_{\mathrm{S}} \otimes \mathbb{1}_{\mathrm{S'}}\big(|\Phi_{\mathrm{SS'}}\rangle\langle\Phi_{\mathrm{SS'}}| \big)$$

Correlation measure for dynamics

Bi-partite system:

$\dim(A) = \dim(B) = d$

Choi-Jamiołkowski Isomorphism



Normalised quantum mutual information of the Choi-Jamiołkowski state:

$$\bar{I}(\mathcal{E}_{\mathrm{S}}) := \frac{I(\rho_{\mathrm{S}}^{\mathrm{CJ}})}{4\log d} := \frac{1}{4\log d}\left(S\left(\rho_{\mathrm{S}}^{\mathrm{CJ}}|_{\mathrm{AA}'}\right) + S\left(\rho_{\mathrm{S}}^{\mathrm{CJ}}|_{\mathrm{BB}'}\right) - S\left(\rho_{\mathrm{S}}^{\mathrm{CJ}}\right)\right)$$

with $\rho_{\mathrm{S}}^{\mathrm{CJ}}|_{\mathrm{AA}'} := \mathrm{Tr}_{\mathrm{BB}'}(\rho_{\mathrm{S}}^{\mathrm{CJ}})$ and $\rho_{\mathrm{S}}^{\mathrm{CJ}}|_{\mathrm{BB}'} := \mathrm{Tr}_{\mathrm{AA}'}(\rho_{\mathrm{S}}^{\mathrm{CJ}})$

Quantifying Correlations in Dynamics

Properties of the quantifier $\bar{I}(\mathcal{E}) := \frac{1}{4\log d} I(\rho^{\mathrm{CJ}})$

☑ Faithfullness: If and only if $I(\rho_{\mathrm{CJ}}) > 0$ ➡ Correlated Dynamics

$$\bar{I}(\mathcal{E}) = 0 \Leftrightarrow \mathcal{E} = \mathcal{E}_{\mathrm{A}} \otimes \mathcal{E}_{\mathrm{B}}$$

☑ Normalisation: $\bar{I}(\mathcal{E}) \in [0, 1]$

$I(\rho_{\mathrm{S}}^{\mathrm{CJ}}) = 2\log d^2$ maximal for a maximally entangled state w/ respect to partition $\mathrm{AA}'|\mathrm{BB}'$

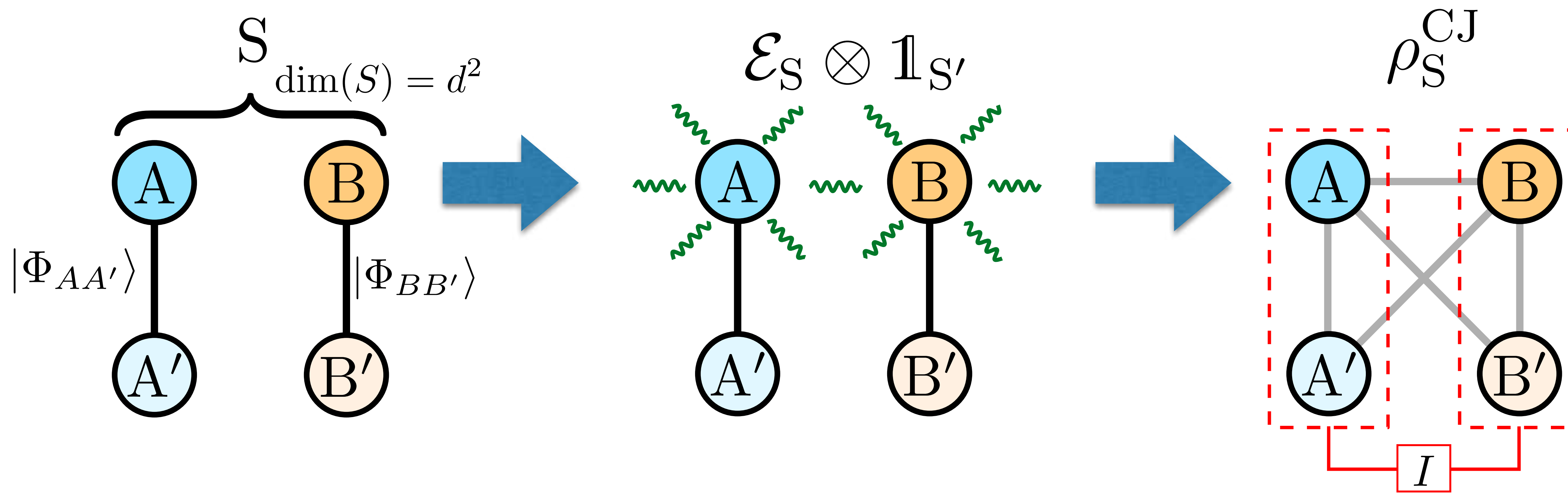
☑ Fundamental Law:

$$\bar{I}(\mathcal{E}) \geq \bar{I}[(\mathcal{L}_{\mathrm{A}} \otimes \mathcal{L}_{\mathrm{B}})\mathcal{E}(\mathcal{R}_{\mathrm{A}} \otimes \mathcal{R}_{\mathrm{B}})]$$

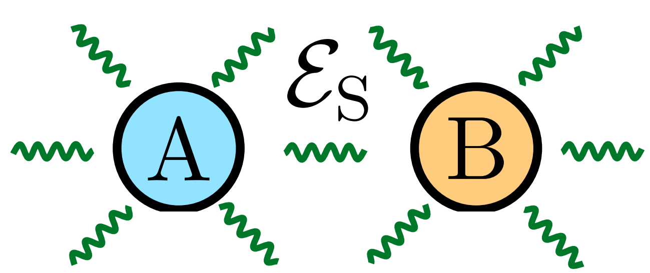
↪ = for local unitaries $U_A \otimes U_B$

Two copies of the quantum system needed?

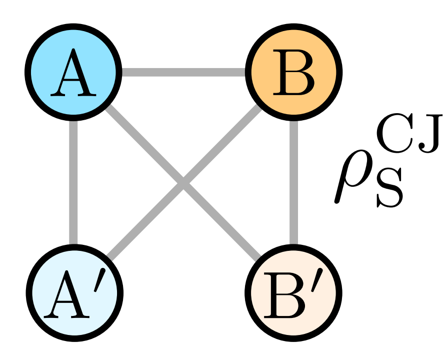
No… mathematical construction only.



Reconstruction of $\mathcal{E}_{\rm S}$
Via quantum <u>process tomography</u>



Equivalent to a quantum
<u>state tomography</u> on $\rho_{\rm S}^{\rm CJ}$



Example: N-qubit-system S $\quad d = 2^N \Rightarrow 4^N(4^N - 1)$ real parameters

N=1: 12, N=2: 240, N=3: 4032, …

Maximally Correlated Dynamics

Resource theory approach

Maximally correlated dynamics cannot be obtained by composing some dynamical map with uncorrelated dynamics

All non-factorable unitaries $U \neq U_A \otimes U_B$ satisfy this criterion!

Not very discriminative

Consider instead:

$$\bar{I}(\mathcal{E}_{\max}) = \max$$

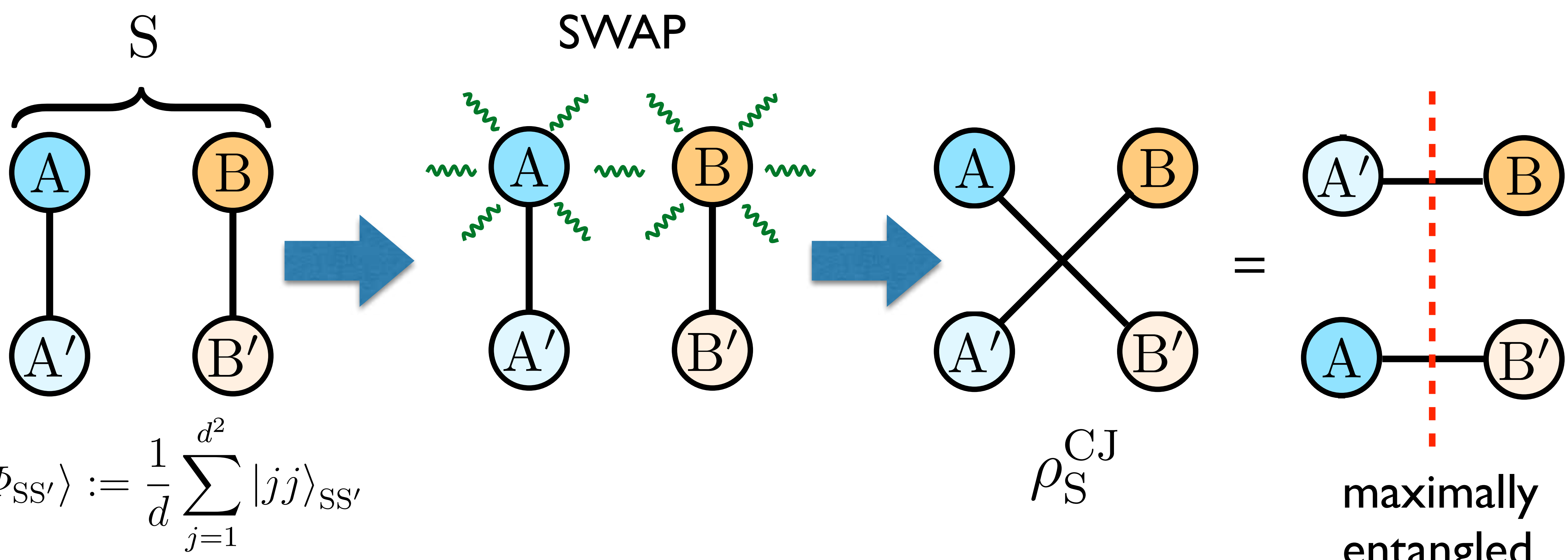
Maximally Correlated Dynamics

Characterization of $\bar{I}(\mathcal{E}_{\max}) = 1$

$$\bar{I}(\mathcal{E}) = 1 \Rightarrow \mathcal{E}(\rho) = U\rho U^\dagger, \quad UU^\dagger = \mathbb{1}$$

$$\mathcal{E}_{\mathrm{S}} \otimes \mathbb{1}_{\mathrm{S}'}\big(|\Phi_{\mathrm{SS}'}\rangle\langle\Phi_{\mathrm{SS}'}| \big) = |\Psi_{(\mathrm{AA}')|(\mathrm{BB}')}\rangle\langle\Psi_{(\mathrm{AA}')|(\mathrm{BB}')}|$$

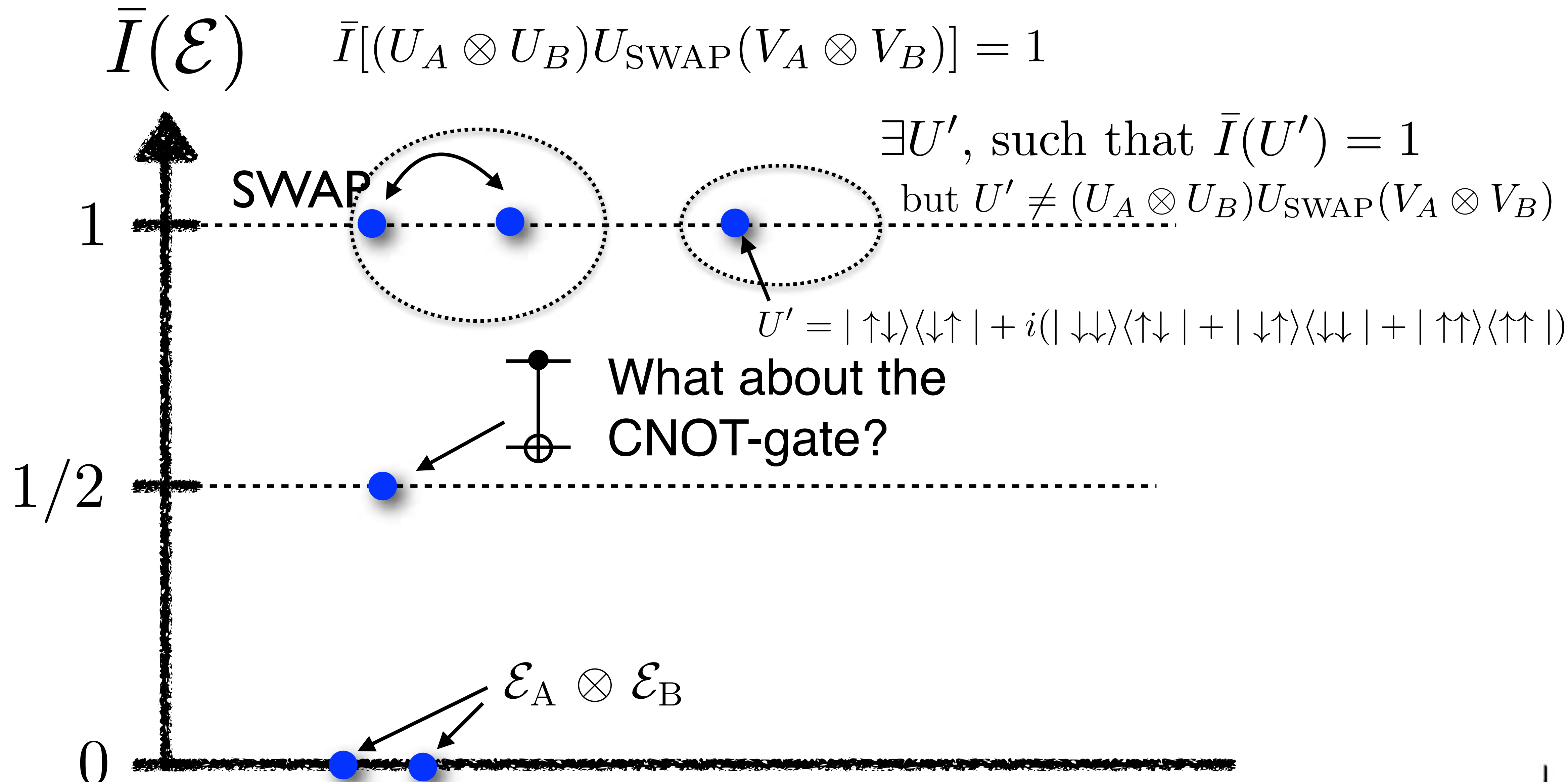
Example: $\bar{I}(U_{\mathrm{SWAP}}) = 1$ *despite the fact that it does not create correlations!*



$$|\Phi_{\mathrm{SS}'}\rangle := \frac{1}{d}\sum_{j=1}^{d^2} |jj\rangle_{\mathrm{SS}'}$$

$\rho_{\mathrm{S}}^{\mathrm{CJ}}$

(Non-)Maximally Correlated Dynamics

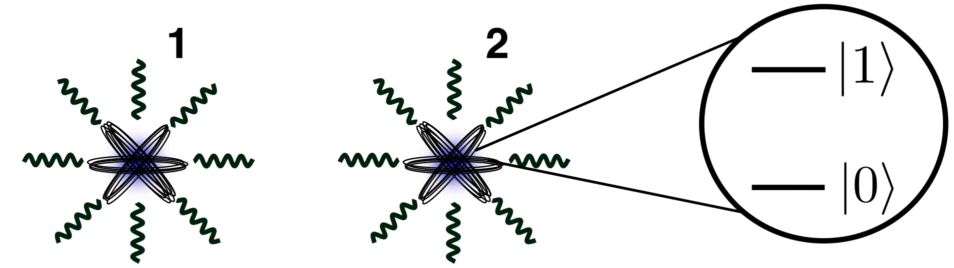


Remark: highly correlating dynamics such as the CNOT gate or maps for dissipative preparation of Bell-states are not maximally correlated according to $\bar{I}$!

Physical Application: Superradiance

Two two-level atoms radiating in the EM vacuum

$$H_S = \frac{\omega_0}{2}\sigma_1^z + \frac{\omega_0}{2}\sigma_2^z$$



$$\frac{d\rho}{dt} = -i[H_S, \rho] + \sum_{j,k=1,2} a_{jk}\left(\sigma_k^- \rho \sigma_j^+ - \tfrac{1}{2}\{\sigma_j^+\sigma_k^-, \rho\}\right)$$

$$a_{jk} \simeq \begin{cases} \gamma_0 \delta_{jk}, & \text{if} \quad r \gg \lambda \\ \gamma_0, & \text{if} \quad r \ll \lambda \end{cases} \qquad \gamma_0 = \tfrac{4}{3}\omega^3 |\boldsymbol{d}|^2$$

Details in the lecture notes

First Application: Superradiance

Two two-level atoms radiating in the EM vacuum

$$H_S = \frac{\omega_0}{2}\sigma_1^z + \frac{\omega_0}{2}\sigma_2^z$$

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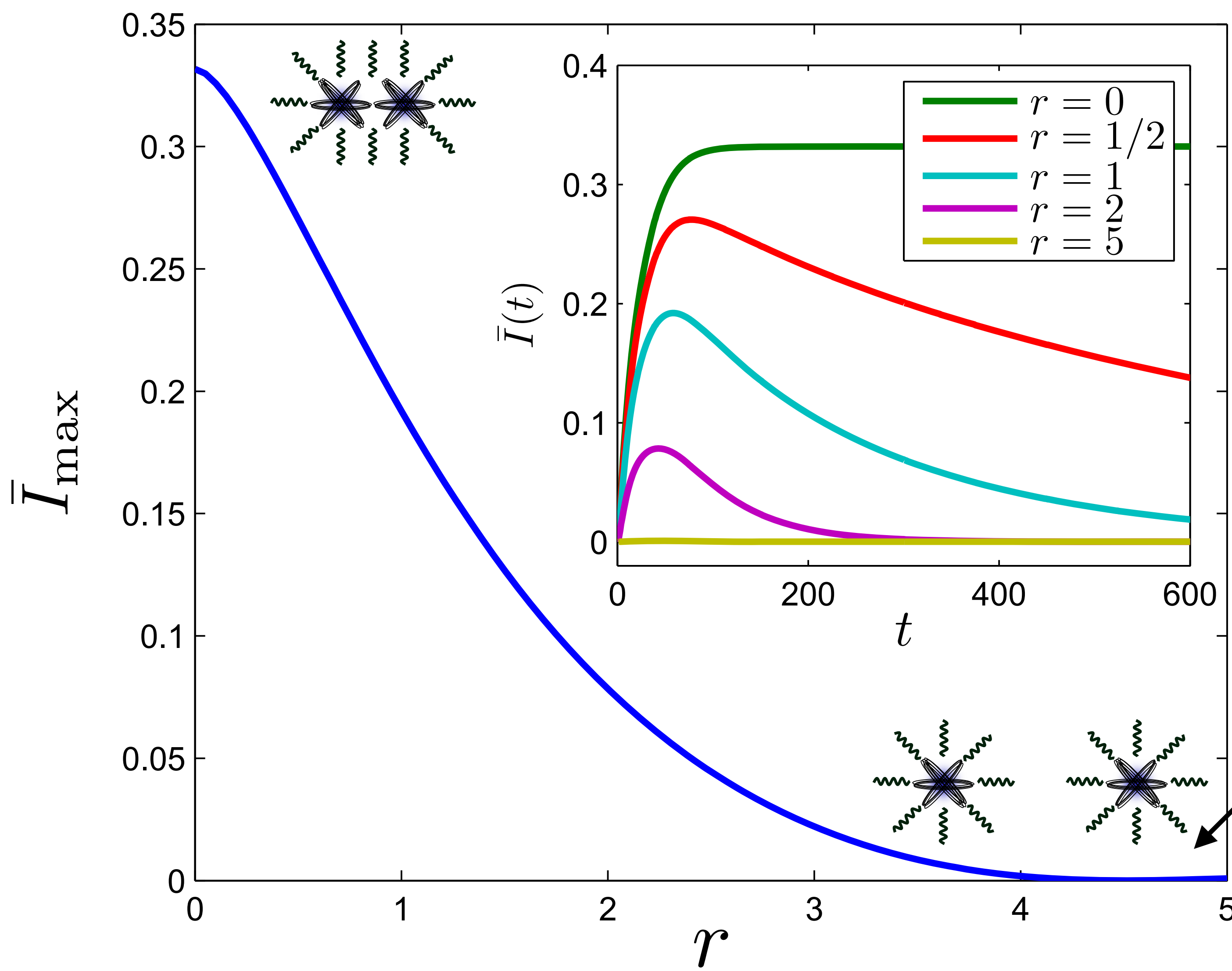
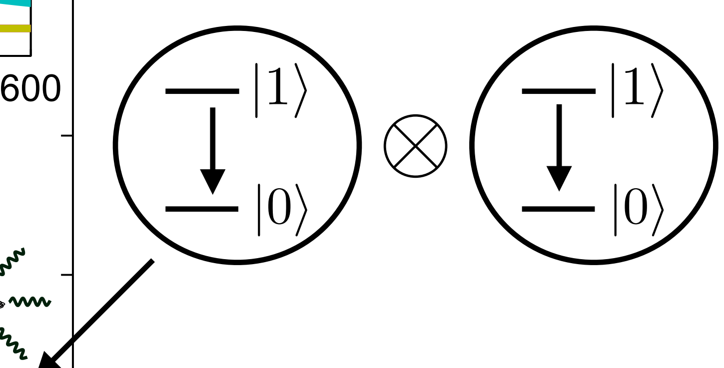


$r \gg \lambda$

Independent decay

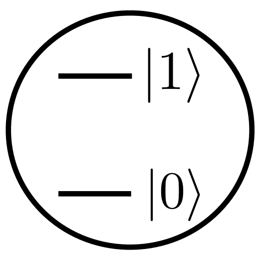


First Application: Superradiance

Two two-level atoms radiating in the EM vacuum

$$H_S = \frac{\omega_0}{2}\sigma_1^z + \frac{\omega_0}{2}\sigma_2^z$$

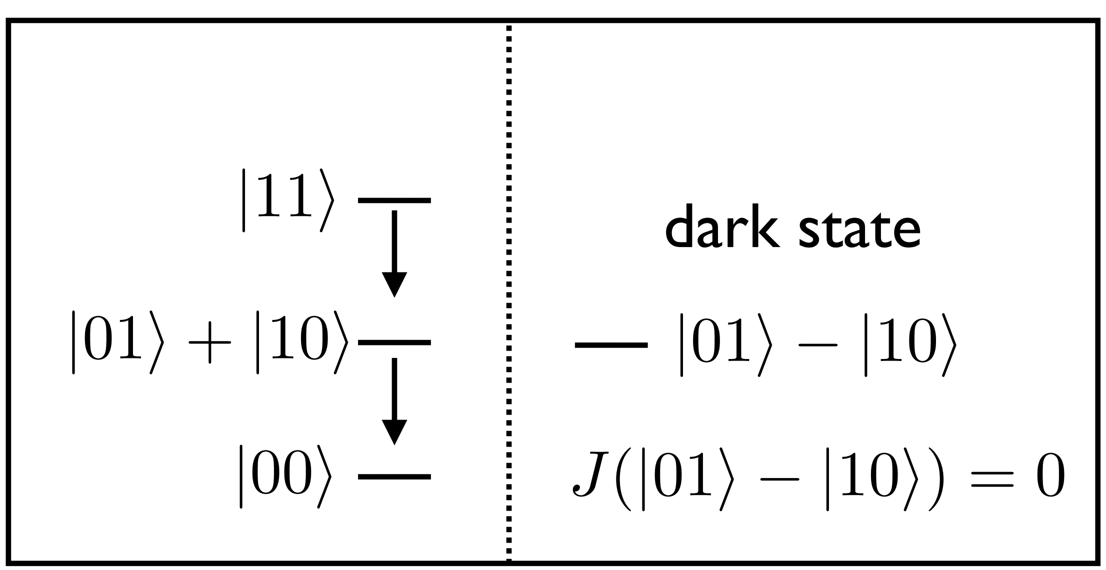
$$\frac{d\rho}{dt} = -i[H_S,\rho] + \sum_{j,k=1,2} a_{jk}\left(\sigma_k^-\rho\sigma_j^+ - \tfrac{1}{2}\{\sigma_j^+\sigma_k^-,\rho\}\right)$$

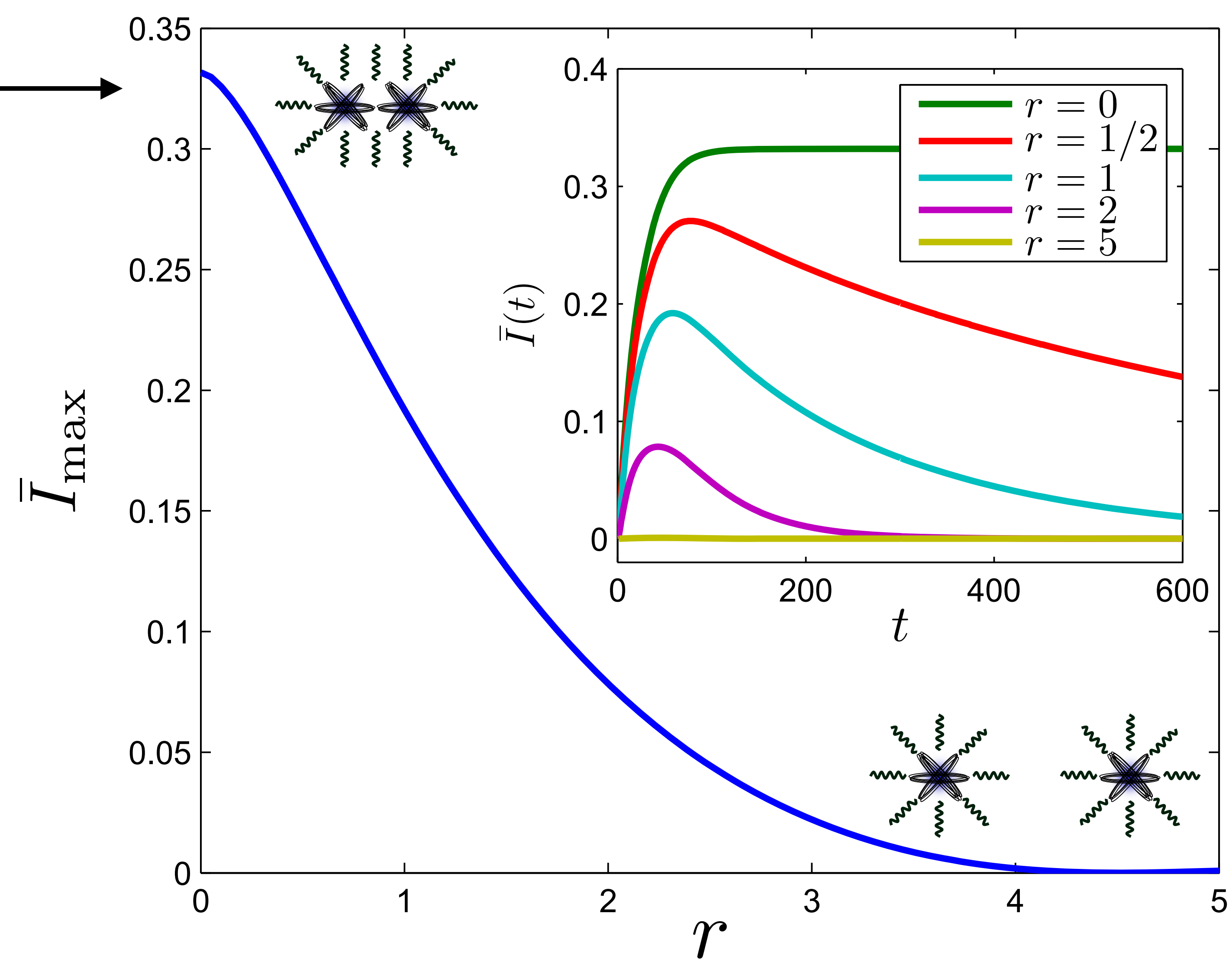


$r \ll \lambda$

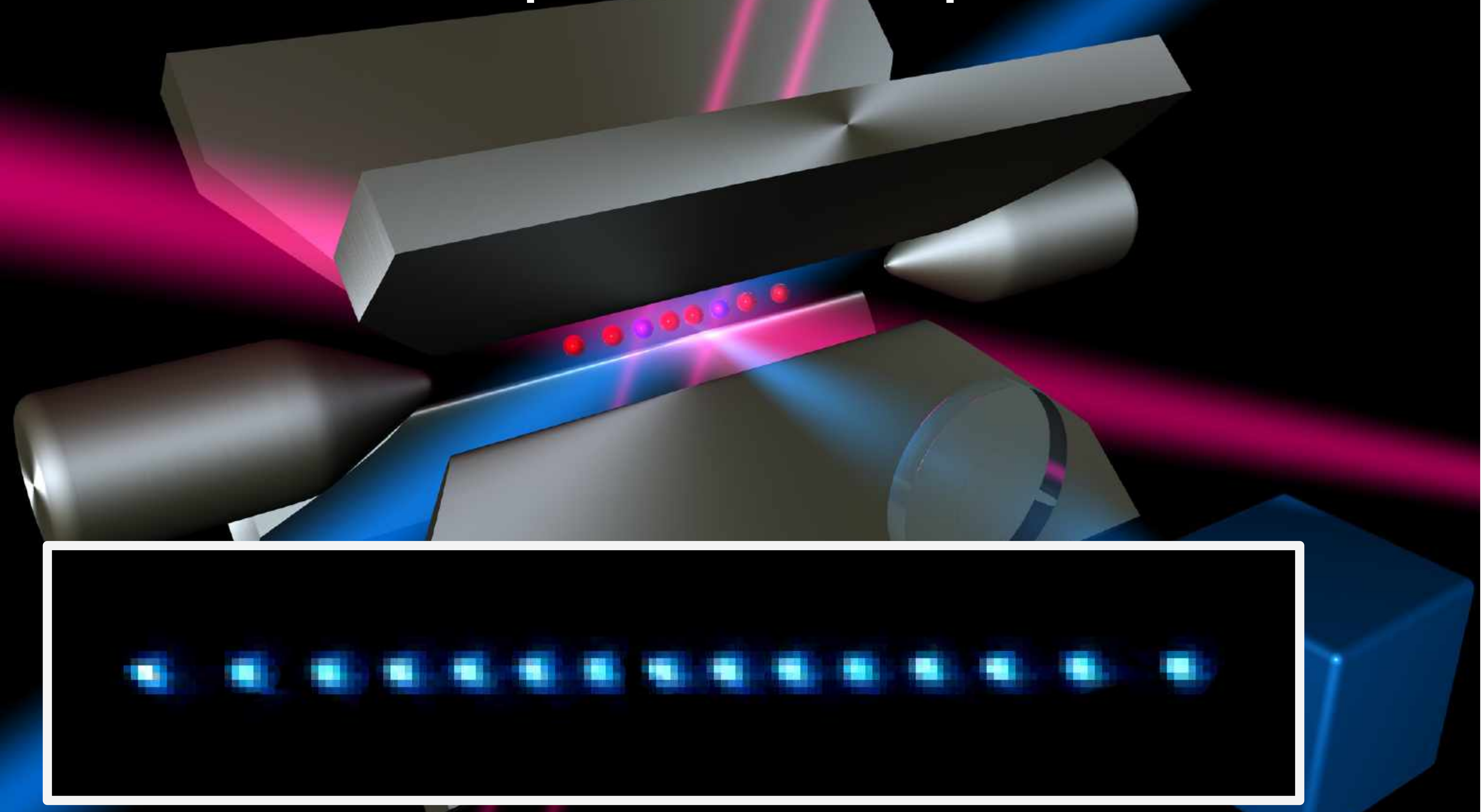
collective jump operator

$J = \sigma_1^- + \sigma_2^-$



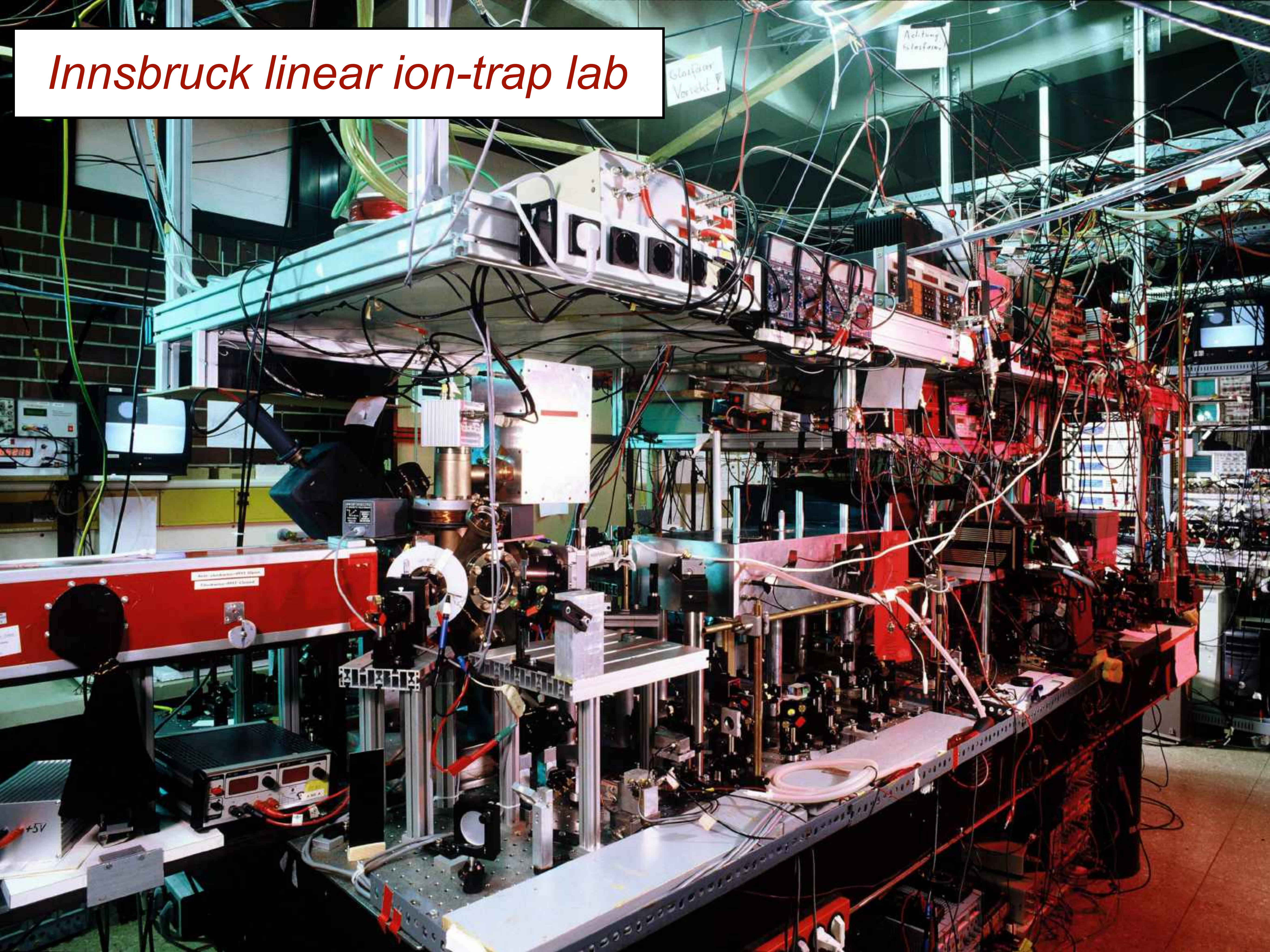


Second application: Noise characterisation in a quantum computer

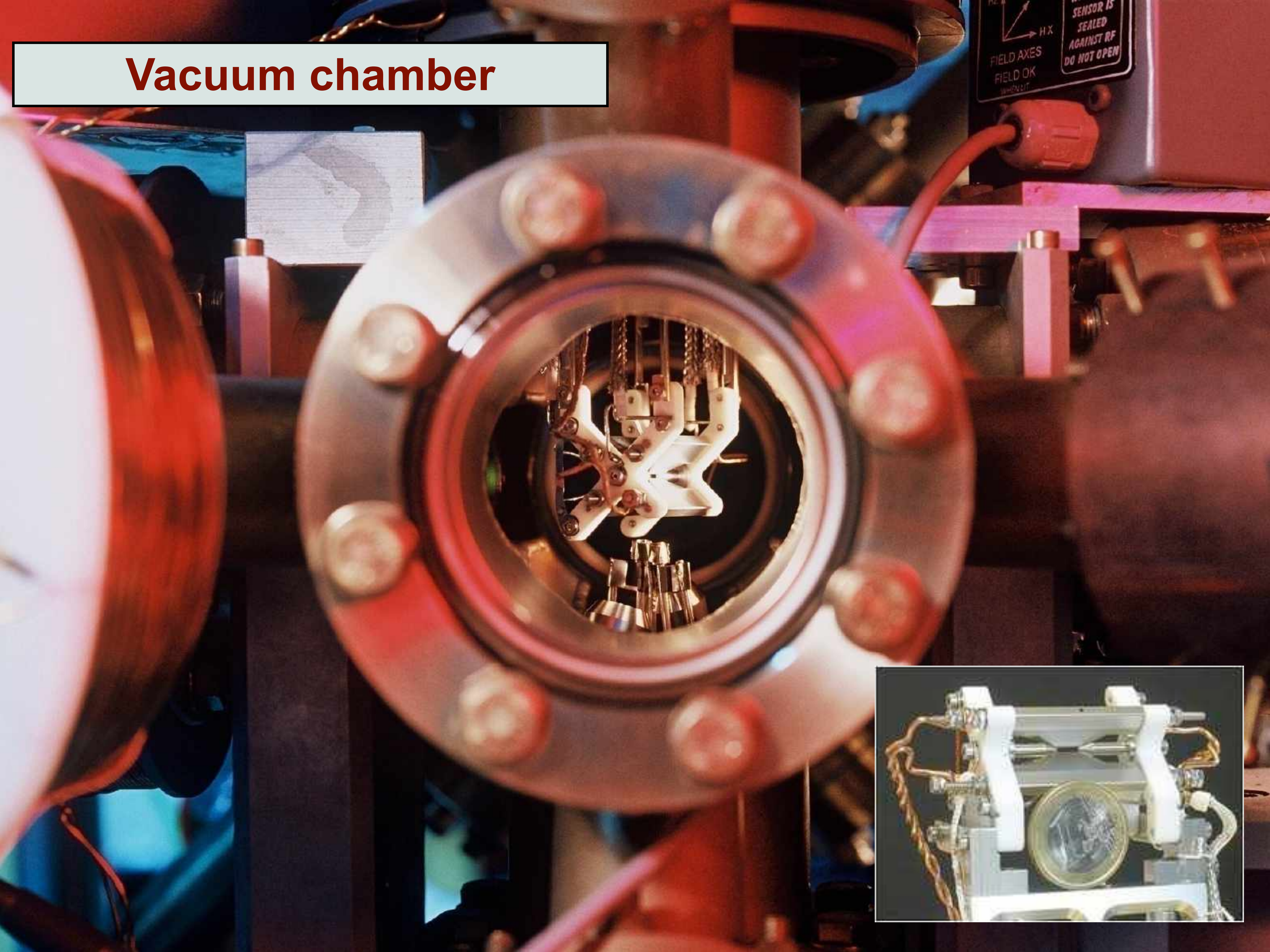


Courtesy: Innsbruck ion-trap group

Innsbruck linear ion-trap lab



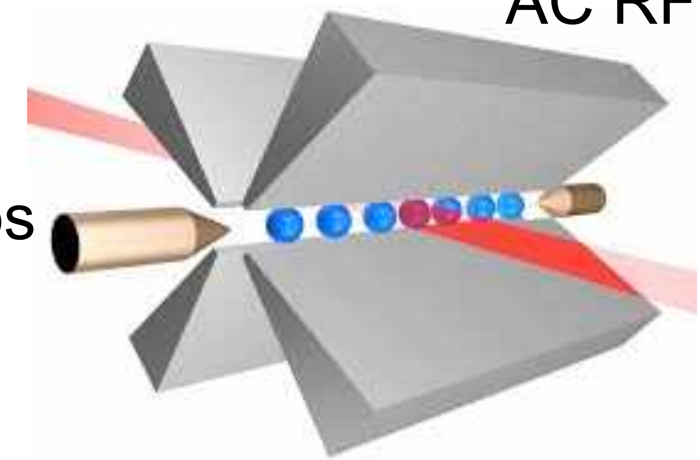
Vacuum chamber

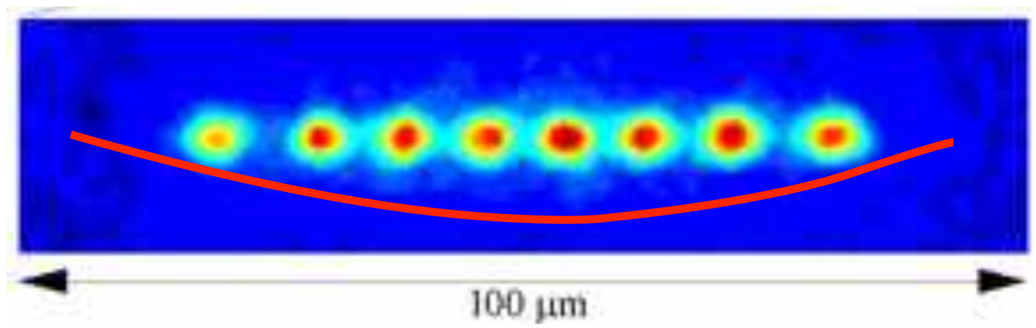


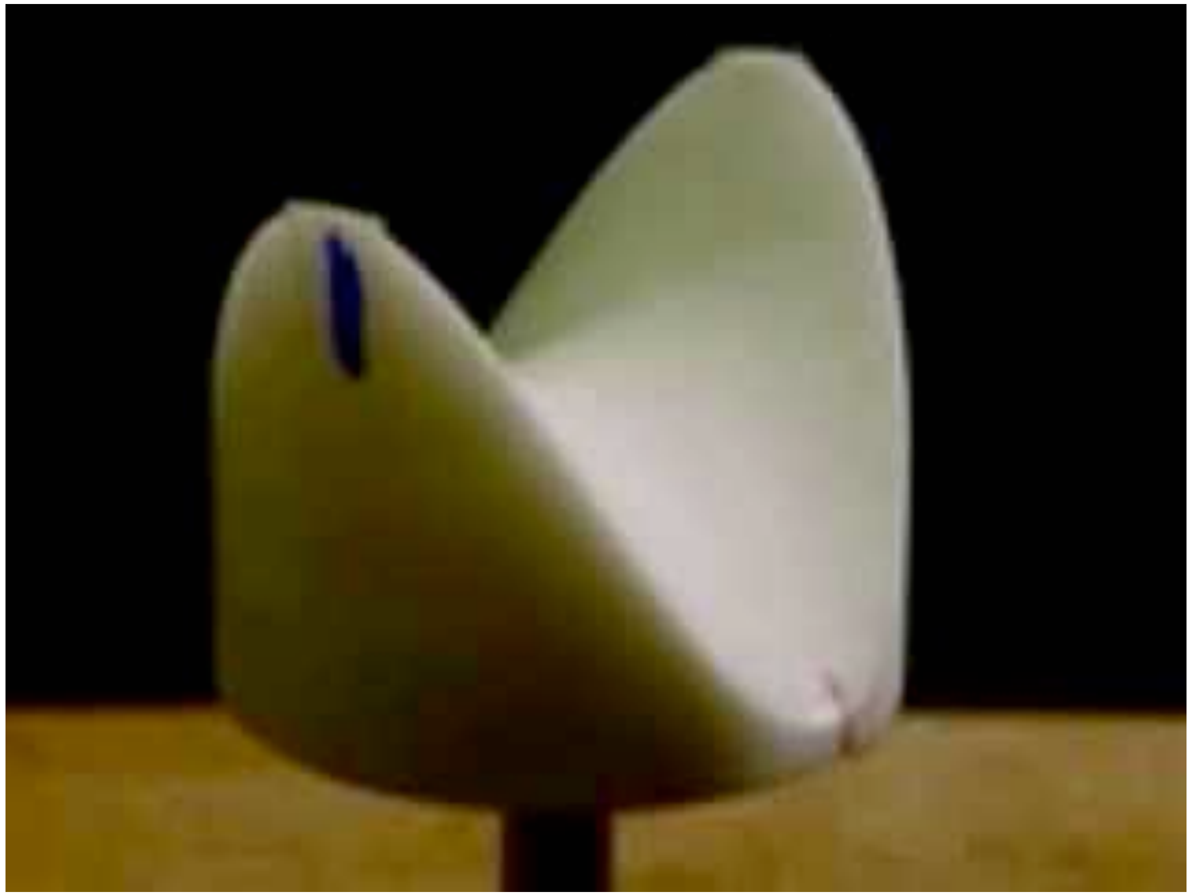
Ions confined in a string by a Paul trap

Ion Coulomb crystals

- Charged particles can't be trapped by static electric fields only (Earnshaw's theorem).
- Solution: **Effective confinement by combining static and oscillating electric fields.**



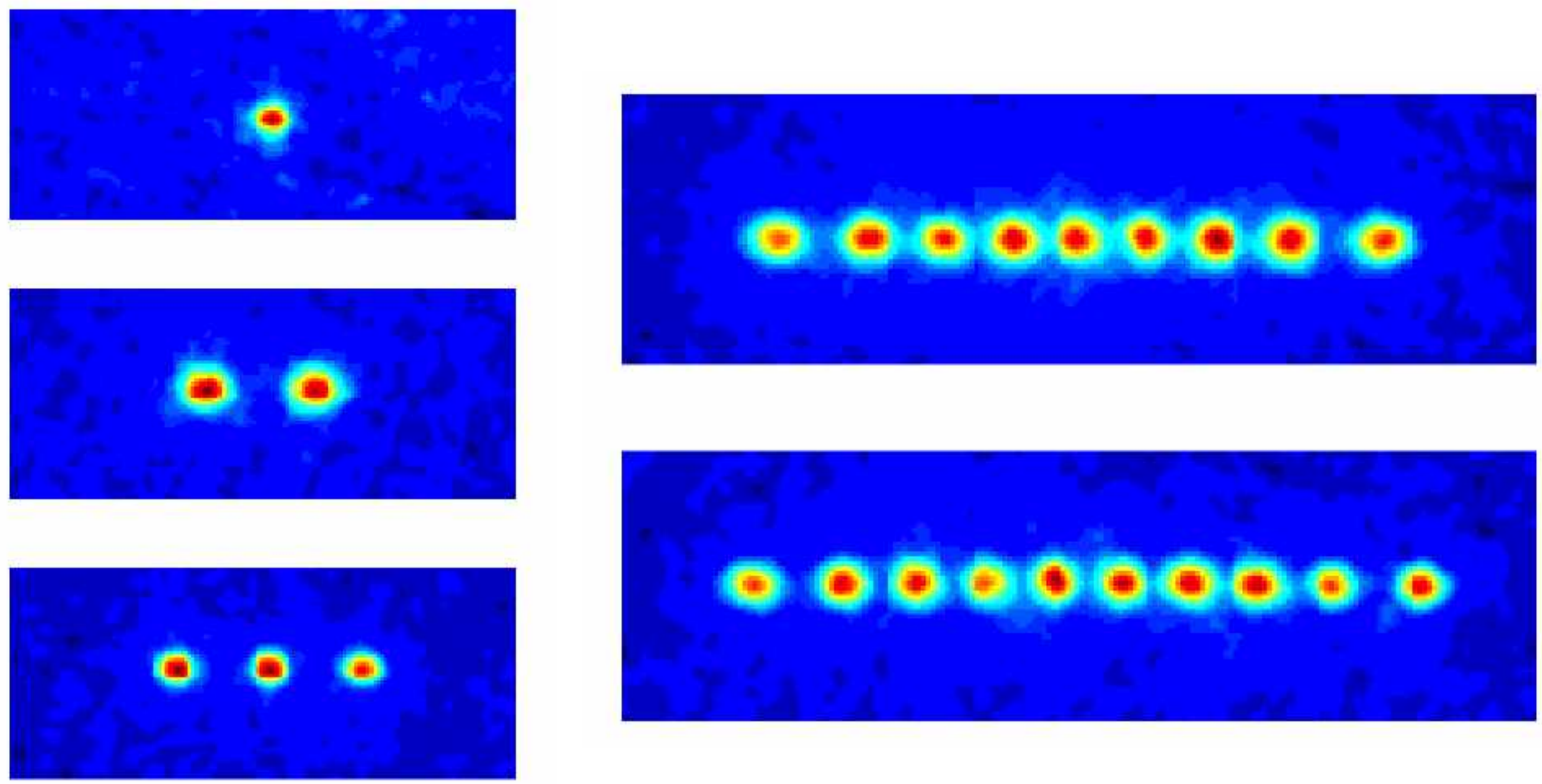




Mechanical analog

- ions form Coulomb crystals (e.g. linear chains)
- laser cooling
- small oscillations around equilibrium positions: collective vibrations (phonons)
- phonons as information bus: entangling quantum gates

Loading your desired number of ions

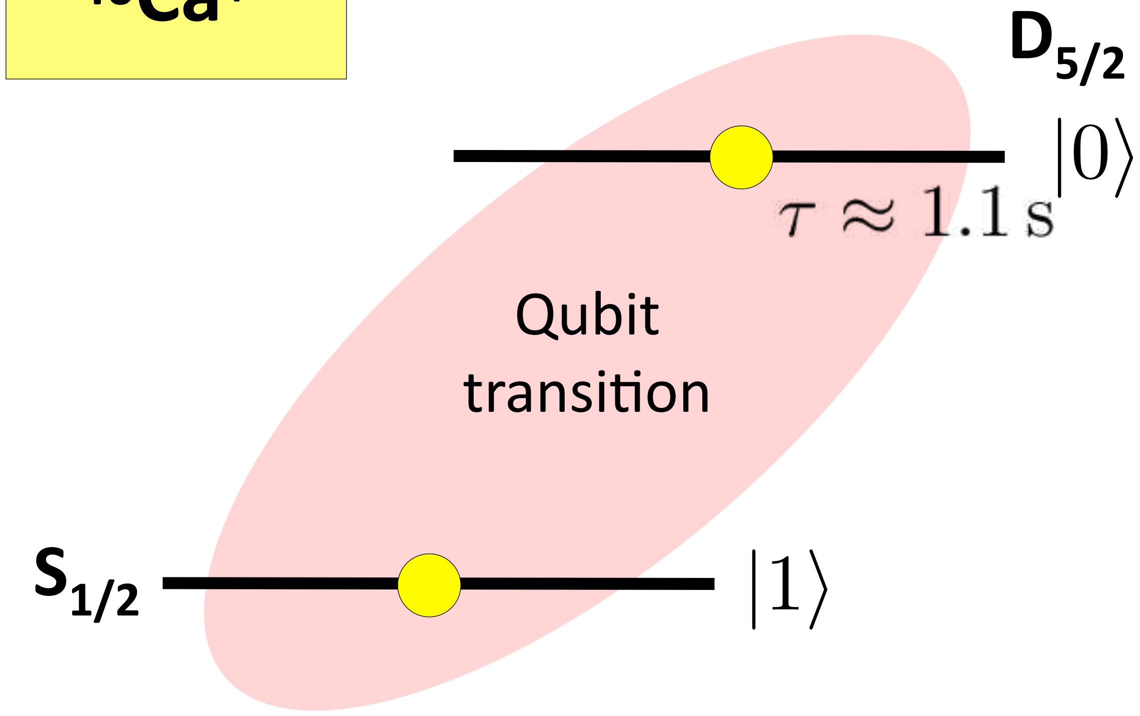


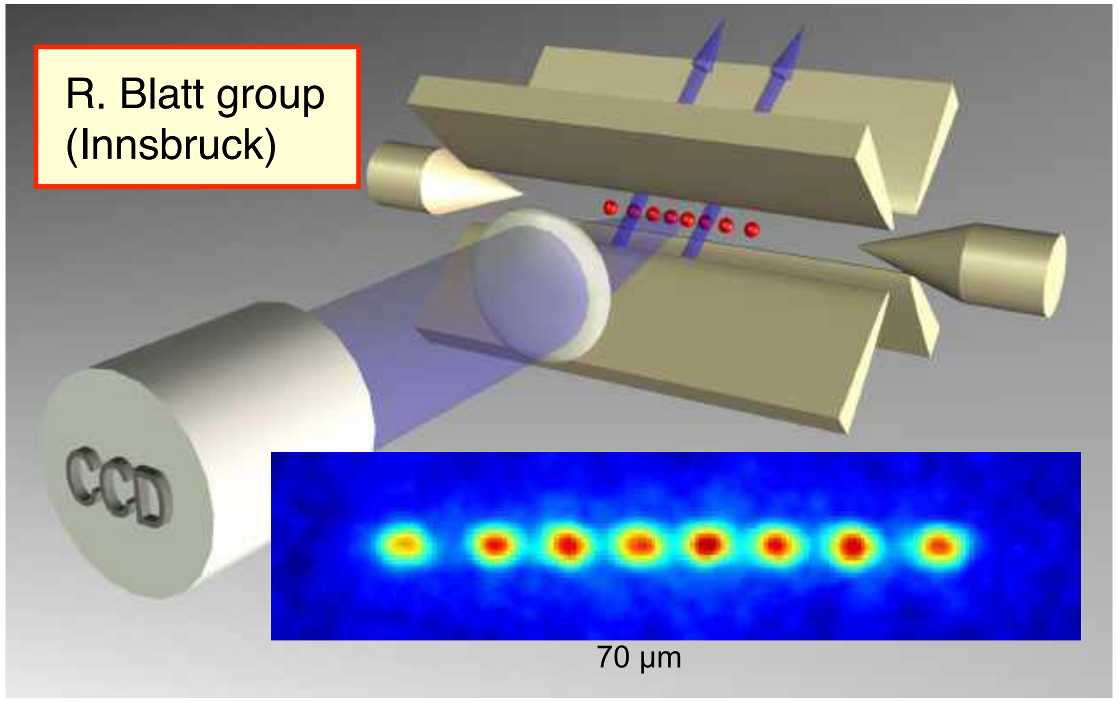
Universität Innsbruck

The qubit register

Simplified electronic level scheme

$^{40}Ca^+$

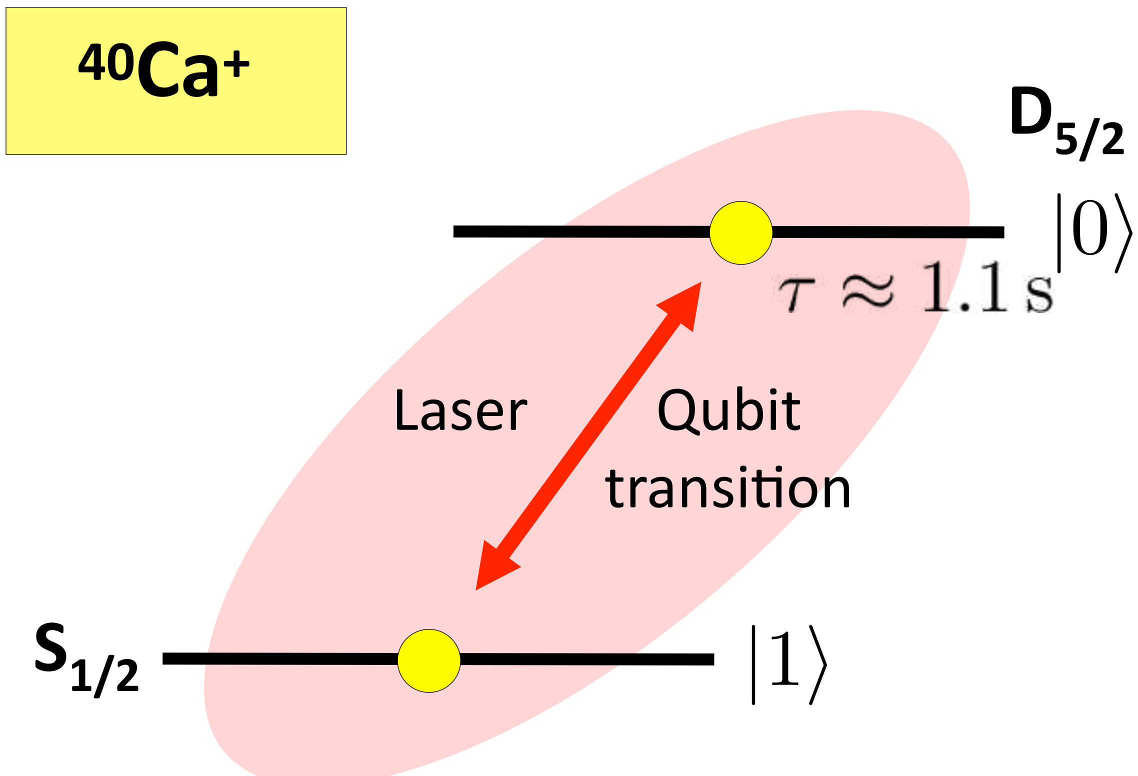


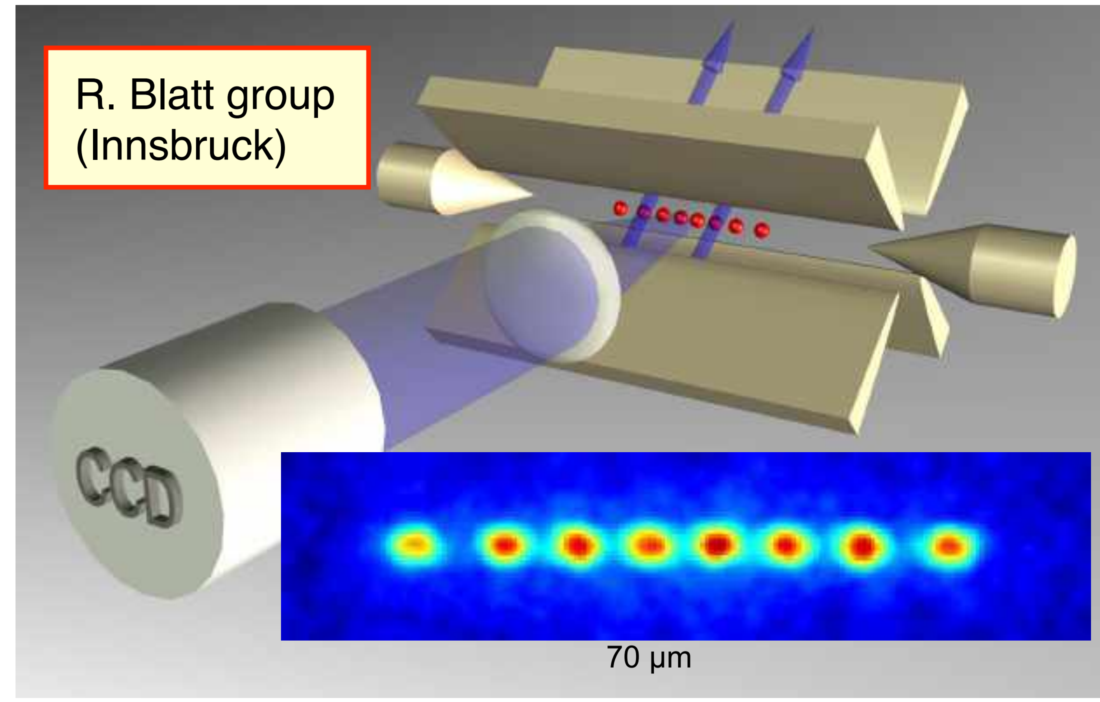


Physical qubits are encoded in (meta-)stable electronic states

Single-qubit gate operations

Simplified electronic level scheme

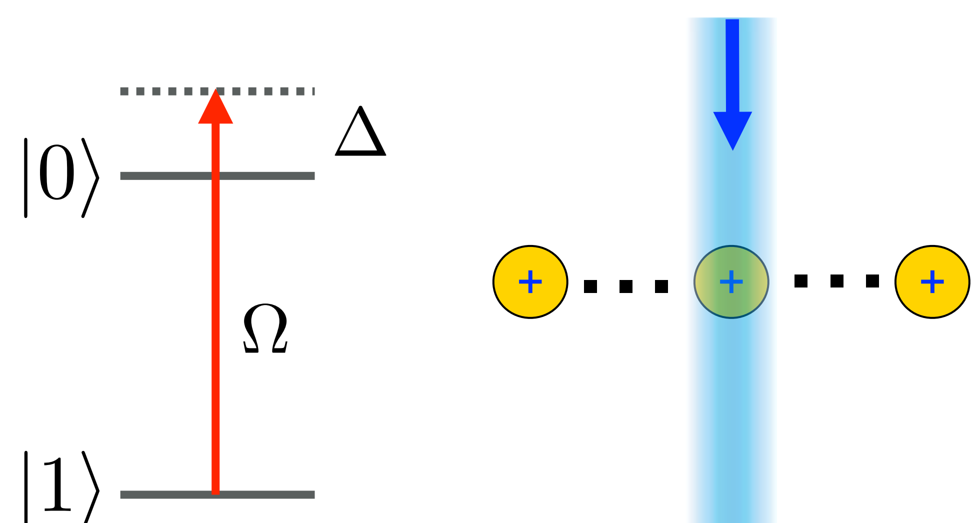




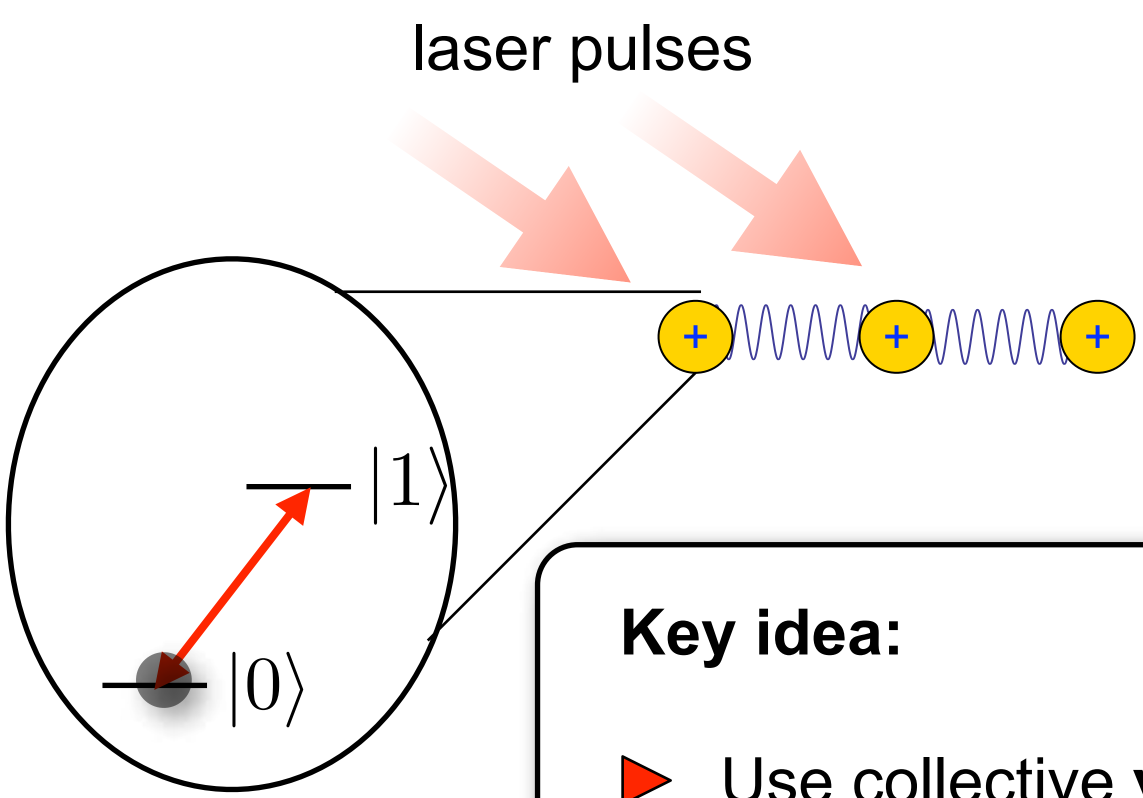
Single-qubit quantum gates

can be realised by tightly focused, near-resonant laser beams applied to individual ions

Example: $U(\phi) = \exp\left(-i\frac{\phi}{2}X\right)$



Entangling gate operations

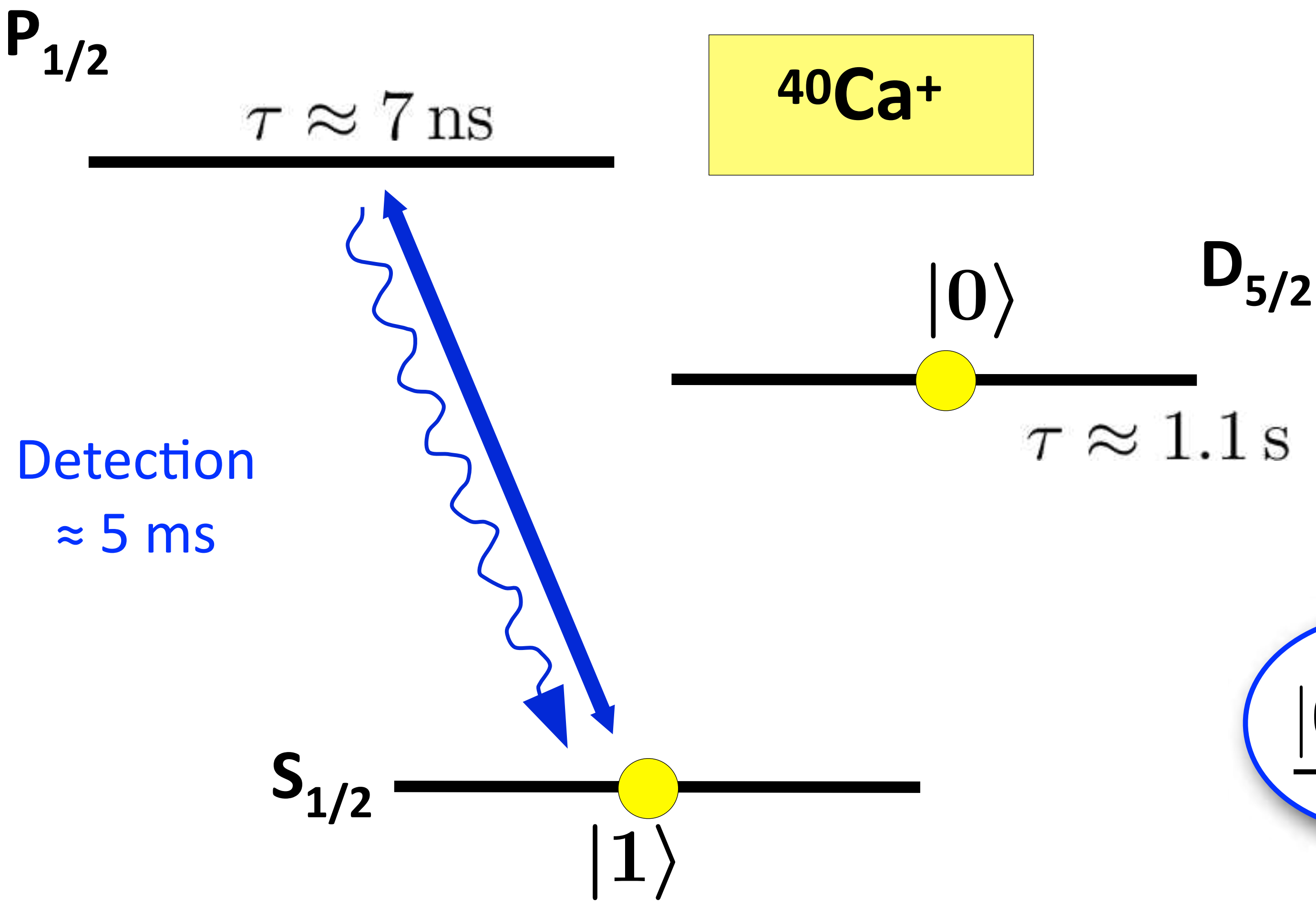


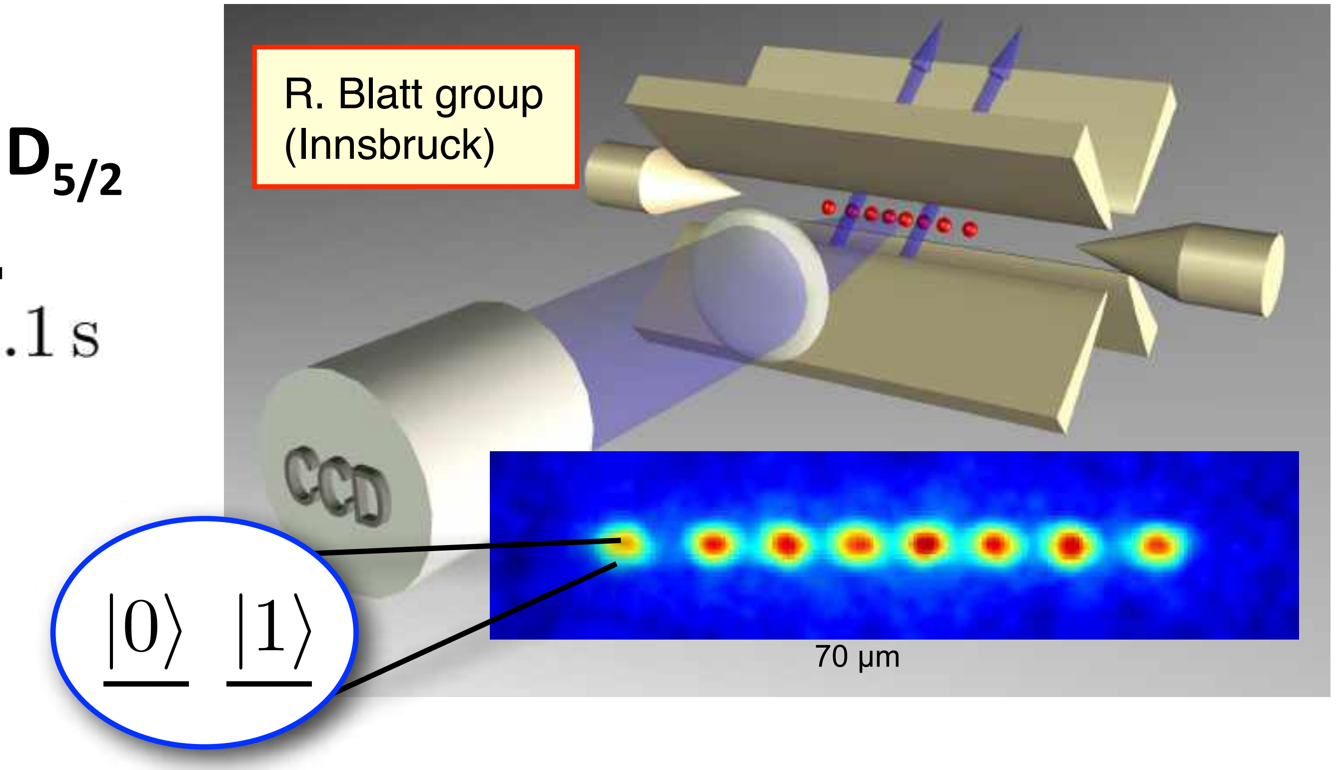
Key idea:

- Use collective **vibrational modes** (phonons) as a **quantum information bus**
- Lasers pulses that **couple electronic (qubit) states** of different ions to the **collective vibrational modes** can **mediate entangling interactions** between two or more qubits
- Two- or multi-ion entangling gates + arbitrary single-qubit rotations = universal set of quantum gates

... fidelity > 99.3 % or higher for 2 qubits, e.g. Benhelm *et al.* Nat. Phys. **4**, 463 (2008)
... 14-qubit entanglement, T. Monz *et al.* PRL **106**,130506 (2011)
... entanglement in a 20-qubit register, Friis *et al.* Phys. Rev. X **8**, 021012 (2018)

Measurement of the qubit register

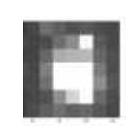




Working principle

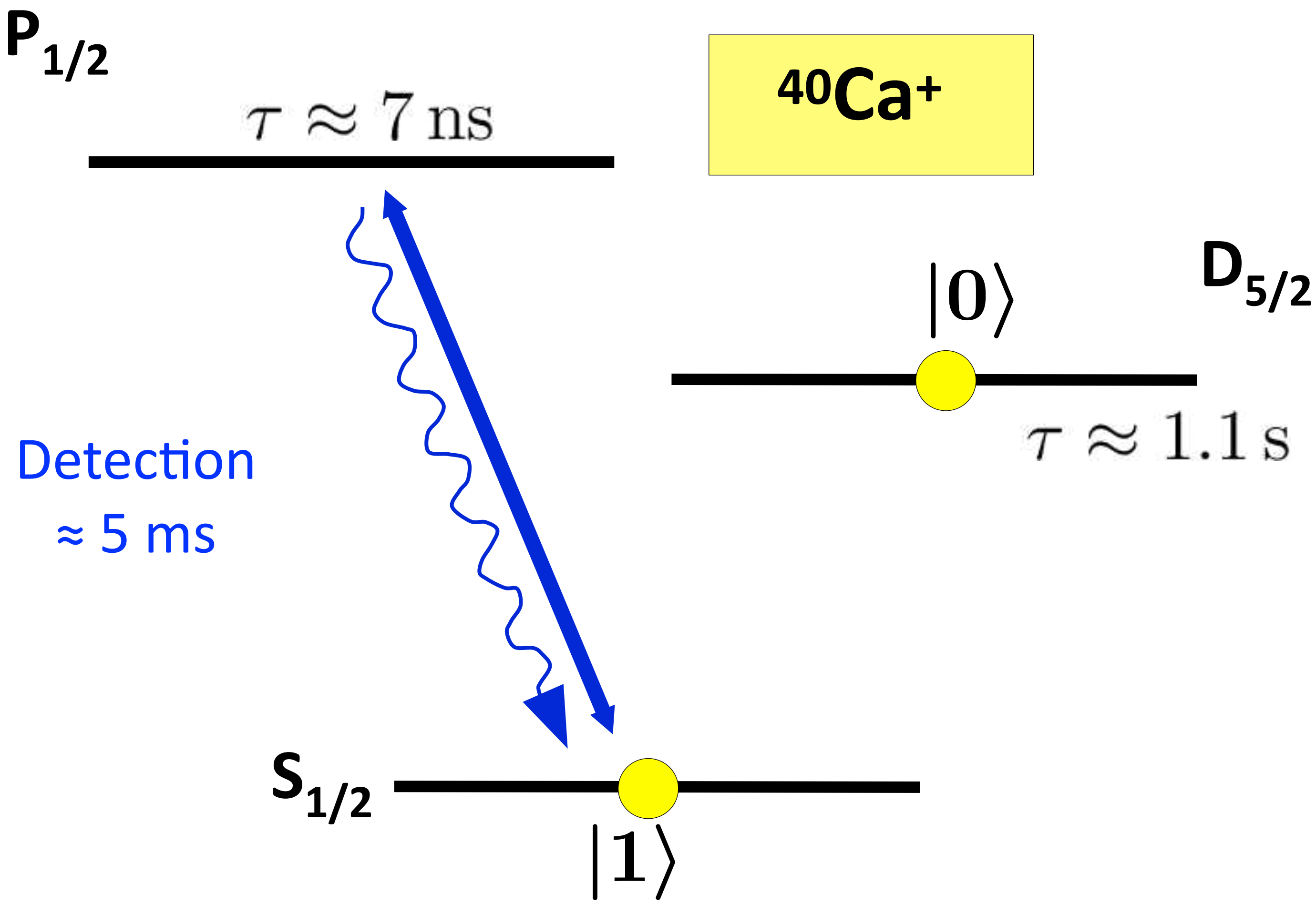
Readout / measurement of qubits: Quantum states can be discriminated via **laser-induced fluorescence** light, recorded by a CCD camera

- Electron in $|1\rangle$ is excited to the P-state, decays quickly, under an emission of a photon, excited again… “cycling transition”
- Electron in $|0\rangle$ does not couple to the laser light.

 bright ion = qubit in $|1\rangle$

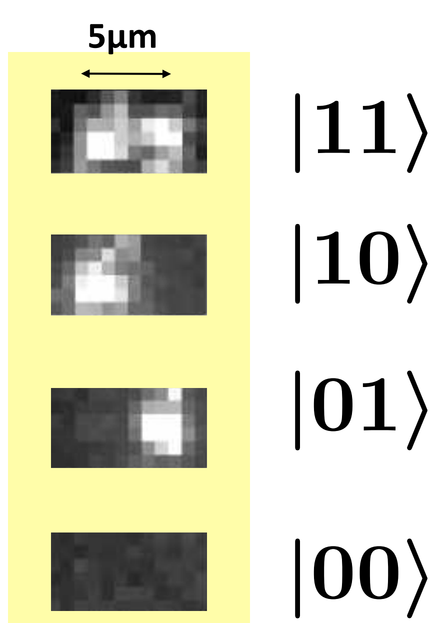
 dark ion = qubit in $|0\rangle$

Quantum state and process reconstruction



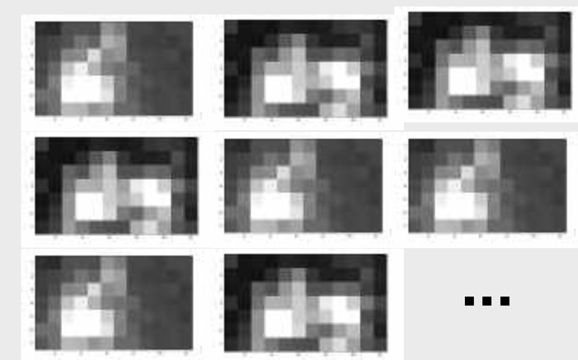
Example: 2 ions

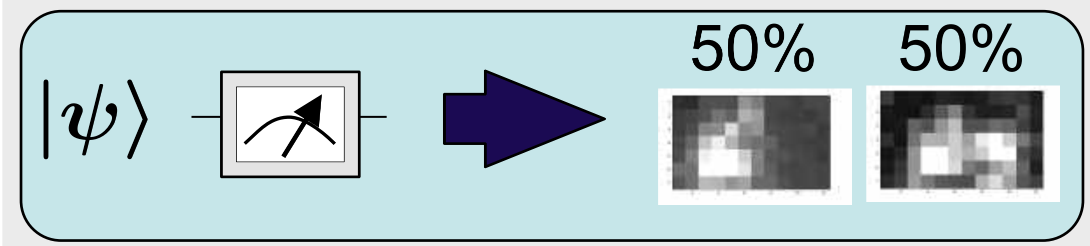
Collect fluorescence with
CCD camera (picture)
Repeat 100-200 times



Measurement of
populations in
$|0\rangle$ and $|1\rangle$

Example, repeat 100 times:





$$|1\rangle \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

or

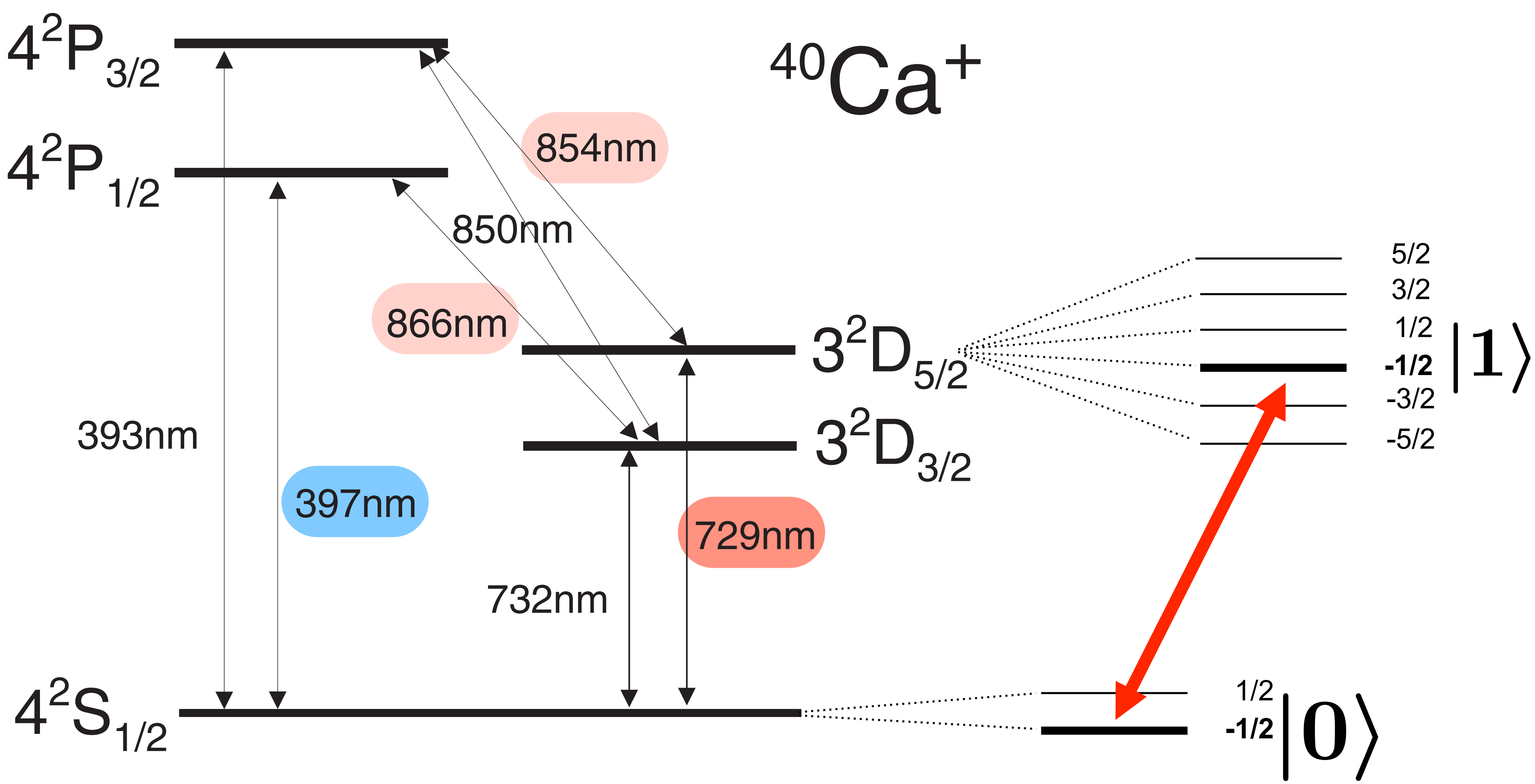
$$|1\rangle \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

?

How do we know which state we have?
Rotate qubits before the measurement

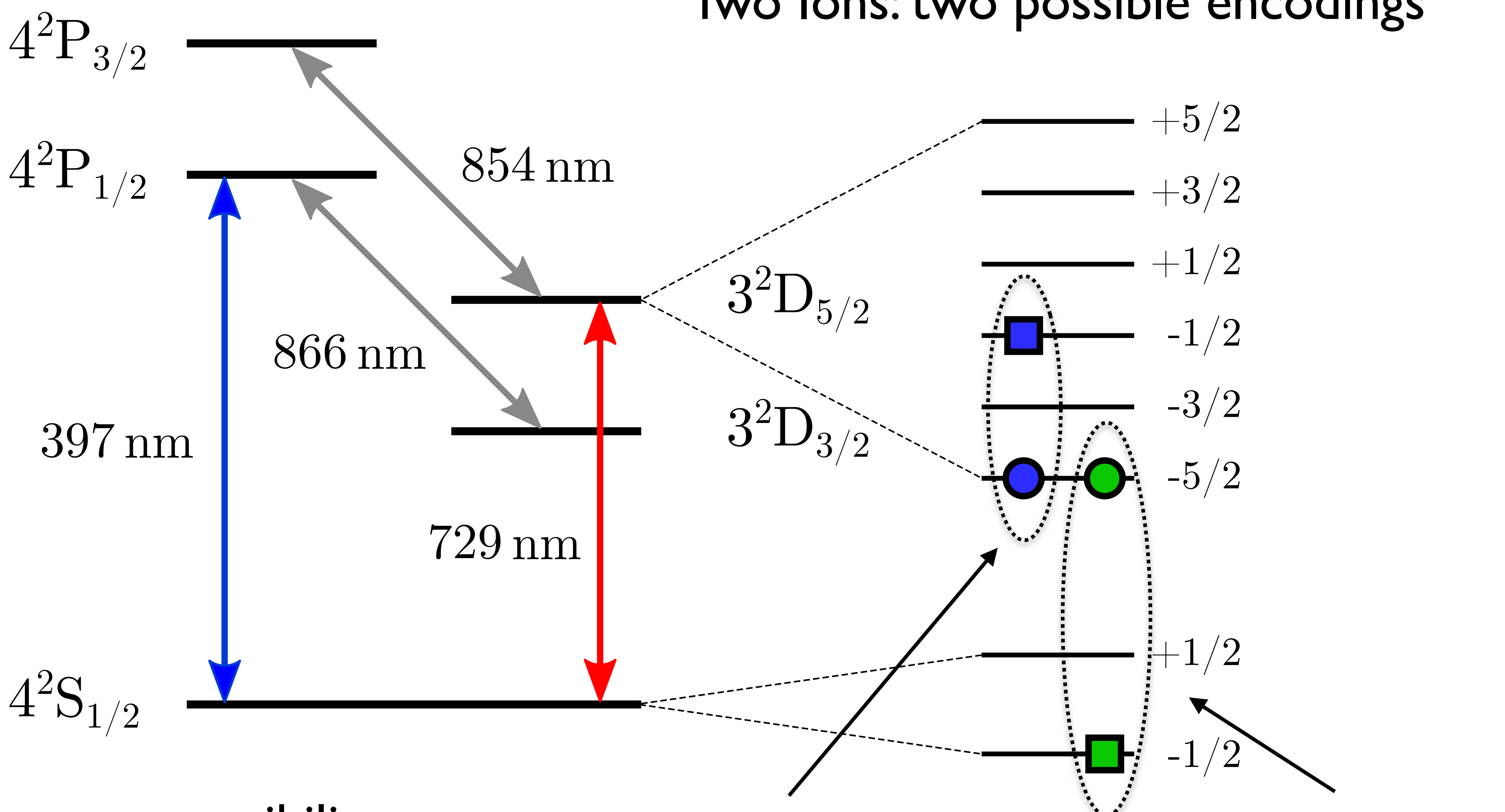
▶ reconstruction of
quantum states and dynamics
(“state and process tomography”)

Characterisation of noise correlations in a real quantum computer



Usual preferred choice: ‘clock qubit’
(First-order insensitive to magnetic field fluctuations)

Two Ions: two possible encodings



Different susceptibility to magnetic field changes

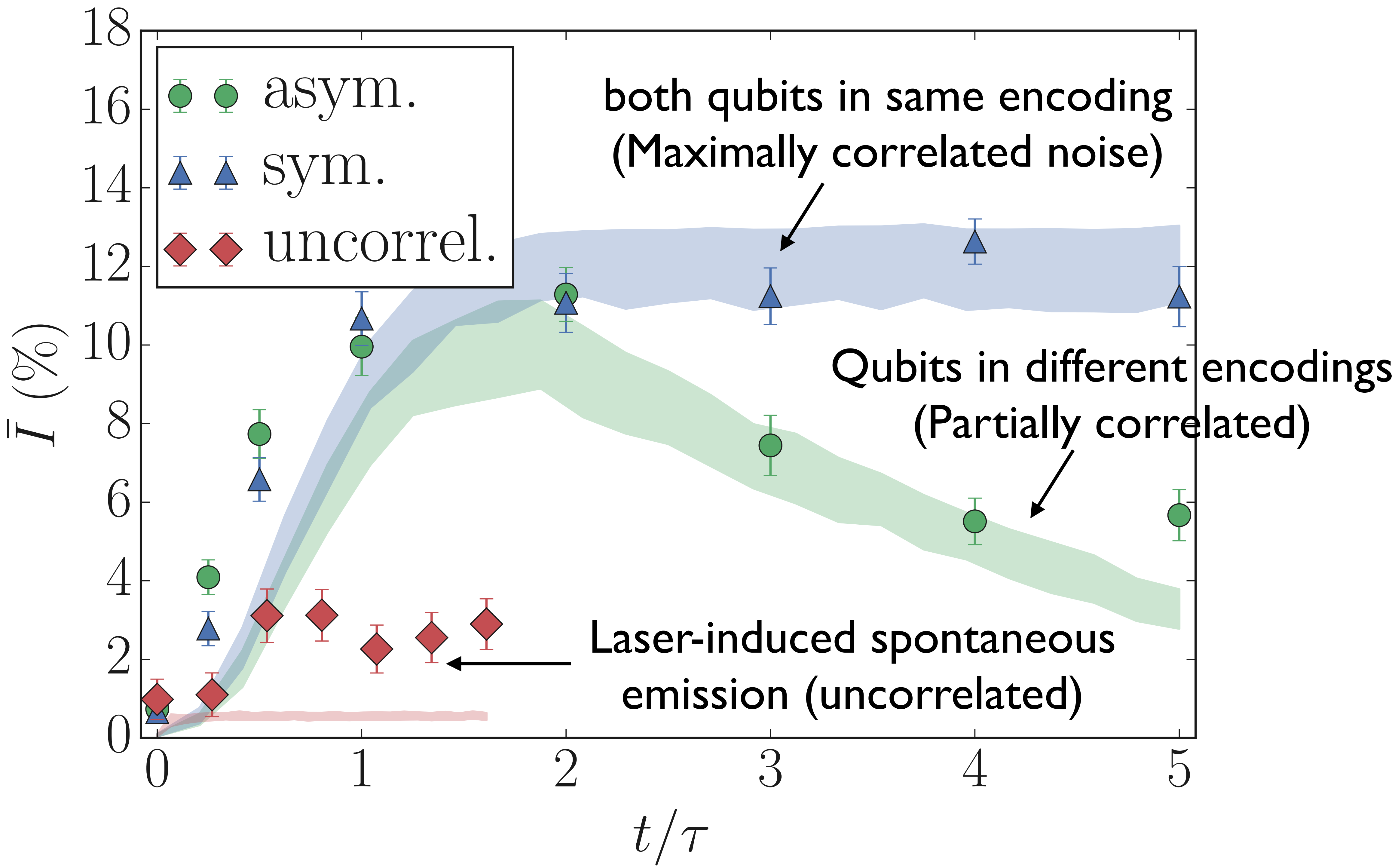
Qubit i accumulates phase

$$\phi_i(t) = \int_0^t d\tau B(\tau)\mu_b g_i$$

Time evolution of a single implementation

$$U(\phi_1) = \exp\left(-i\phi_1(\sigma_1^z + g\sigma_2^z)\right)$$

with the ratio of the Landé factors $g = g_2/g_1$



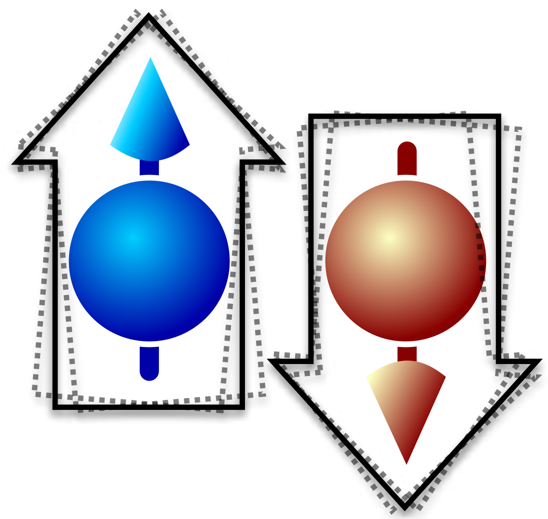
Harnessing spatially correlated noise

Decoherence-free subspaces

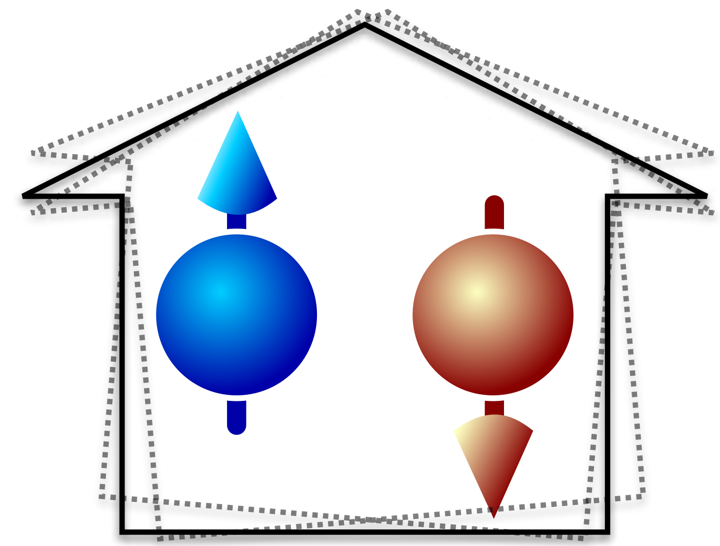
$$H_G(t) = \frac{1}{2} B(t) \sum_k Z_k$$

Random $B(t)$

uncorrelated



Fully correlated



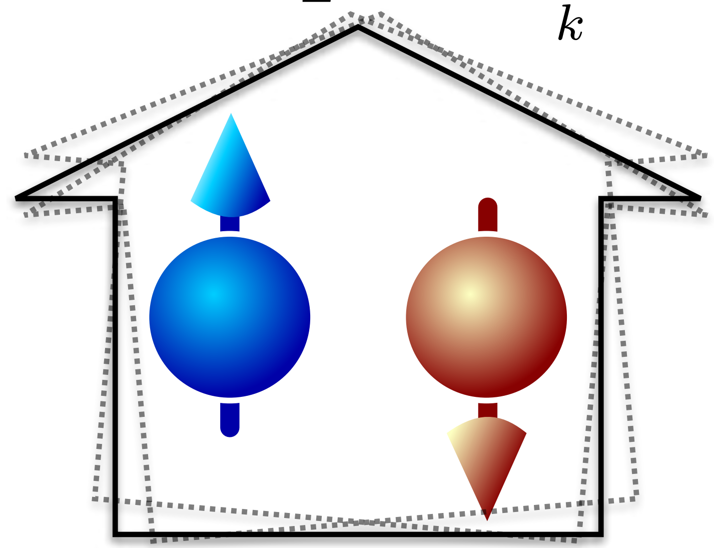
	uncorrelated	Fully correlated
$\|\downarrow\downarrow\rangle$ →	$\|\downarrow\downarrow\rangle$	$\|\downarrow\downarrow\rangle$
$\|\uparrow\downarrow\rangle$ →	$e^{i\phi_1}\|\uparrow\downarrow\rangle$	$e^{i\phi}\|\uparrow\downarrow\rangle$
$\|\downarrow\uparrow\rangle$ →	$e^{i\phi_2}\|\downarrow\uparrow\rangle$	$e^{i\phi}\|\downarrow\uparrow\rangle$
$\|\uparrow\uparrow\rangle$ →	$e^{i(\phi_1+\phi_2)}\|\uparrow\uparrow\rangle$	$e^{2i\phi}\|\uparrow\uparrow\rangle$

Harnessing spatially correlated noise

Decoherence-free subspaces

$$H_G(t) = \frac{1}{2} B(t) \sum_k Z_k$$

Random $B(t)$



take 2 physical qubits

for 1 protected 'logical' qubit

$$| \downarrow\downarrow \rangle$$

$$\left.\begin{array}{l} e^{i\phi} | \uparrow\downarrow \rangle \\ e^{i\phi} | \downarrow\uparrow \rangle \end{array}\right\} \;\Rightarrow\; a | \uparrow\downarrow \rangle + b | \downarrow\uparrow \rangle \;\Rightarrow\; e^{i\phi} (a \overbrace{| \uparrow\downarrow \rangle}^{|0\rangle_L} + b \overbrace{| \downarrow\uparrow \rangle}^{|1\rangle_L})$$

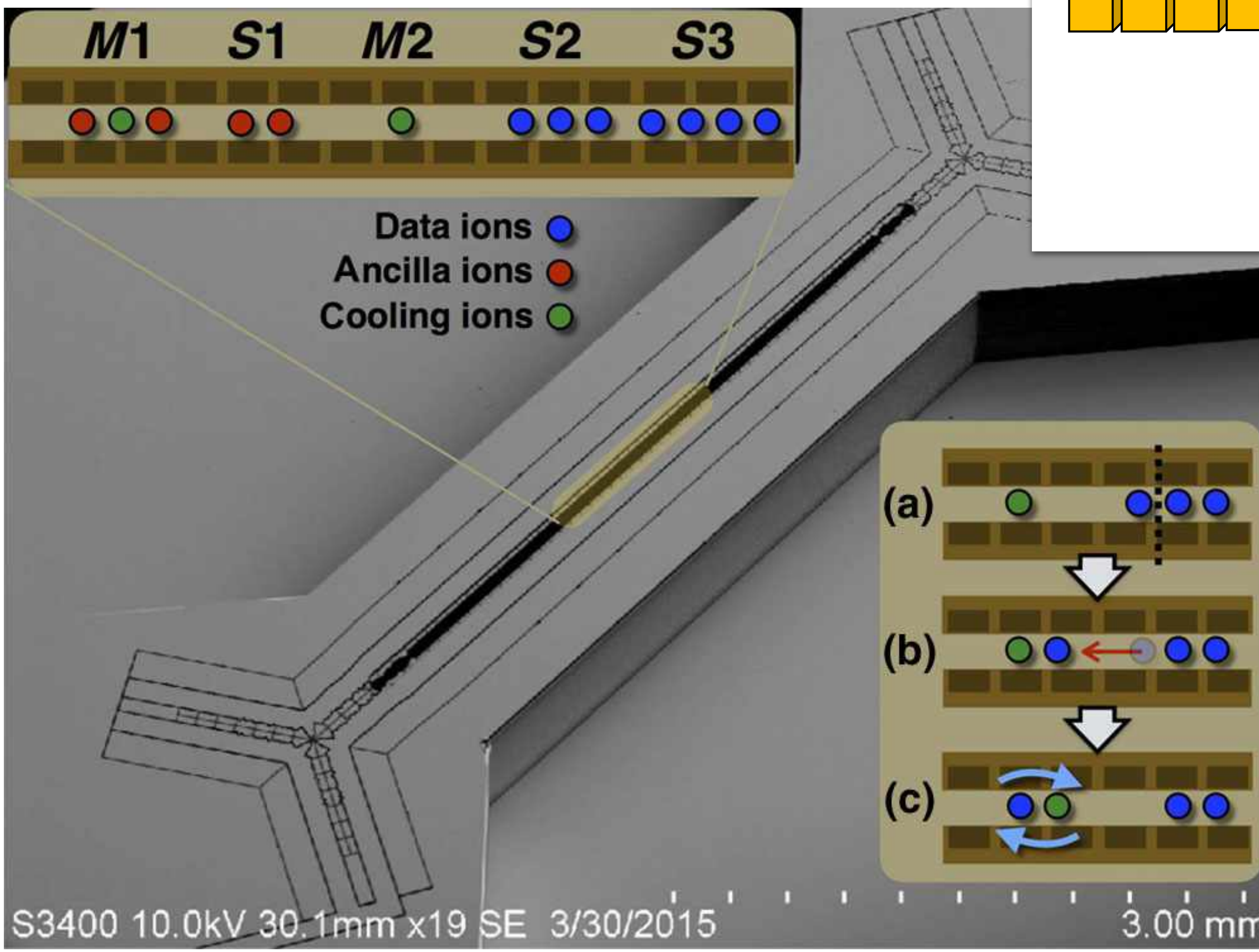
$$e^{2i\phi} | \uparrow\uparrow \rangle$$

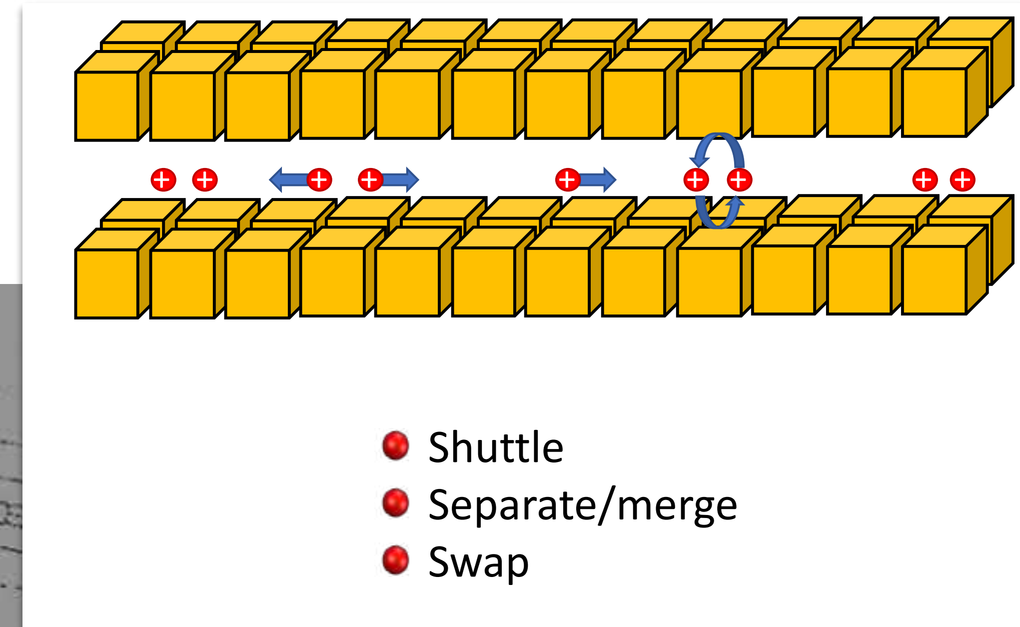
invariant

Decoherence Free Subspace

Harnessing spatially correlated noise

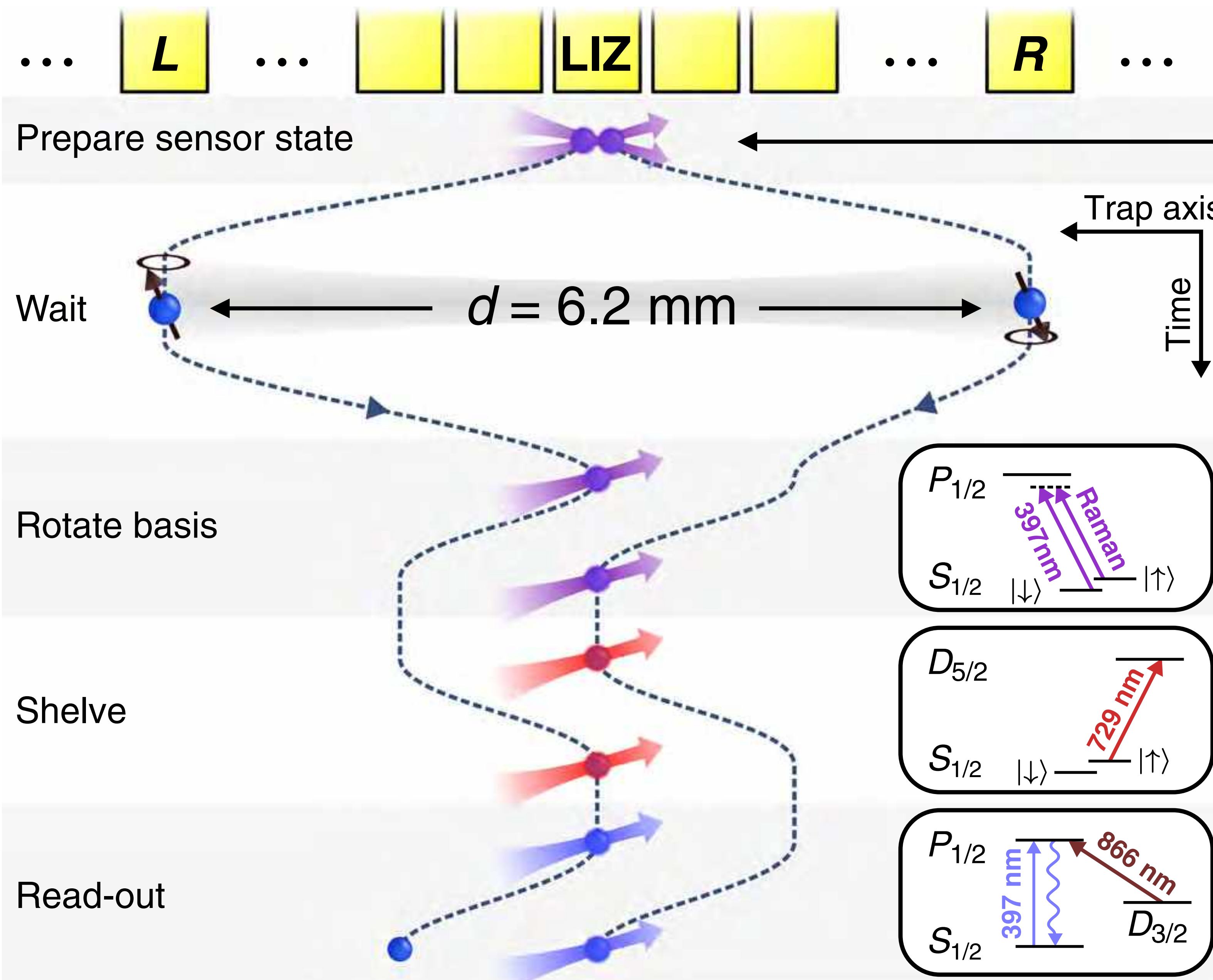
Entanglement-based magnetometry in a trapped-ion quantum computer





Harnessing spatially correlated noise

Entanglement-based magnetometry



Bell state as sensor

$$\frac{1}{\sqrt{2}}(|1\rangle|0\rangle + |0\rangle|1\rangle)$$

magn. field inhomogeneities

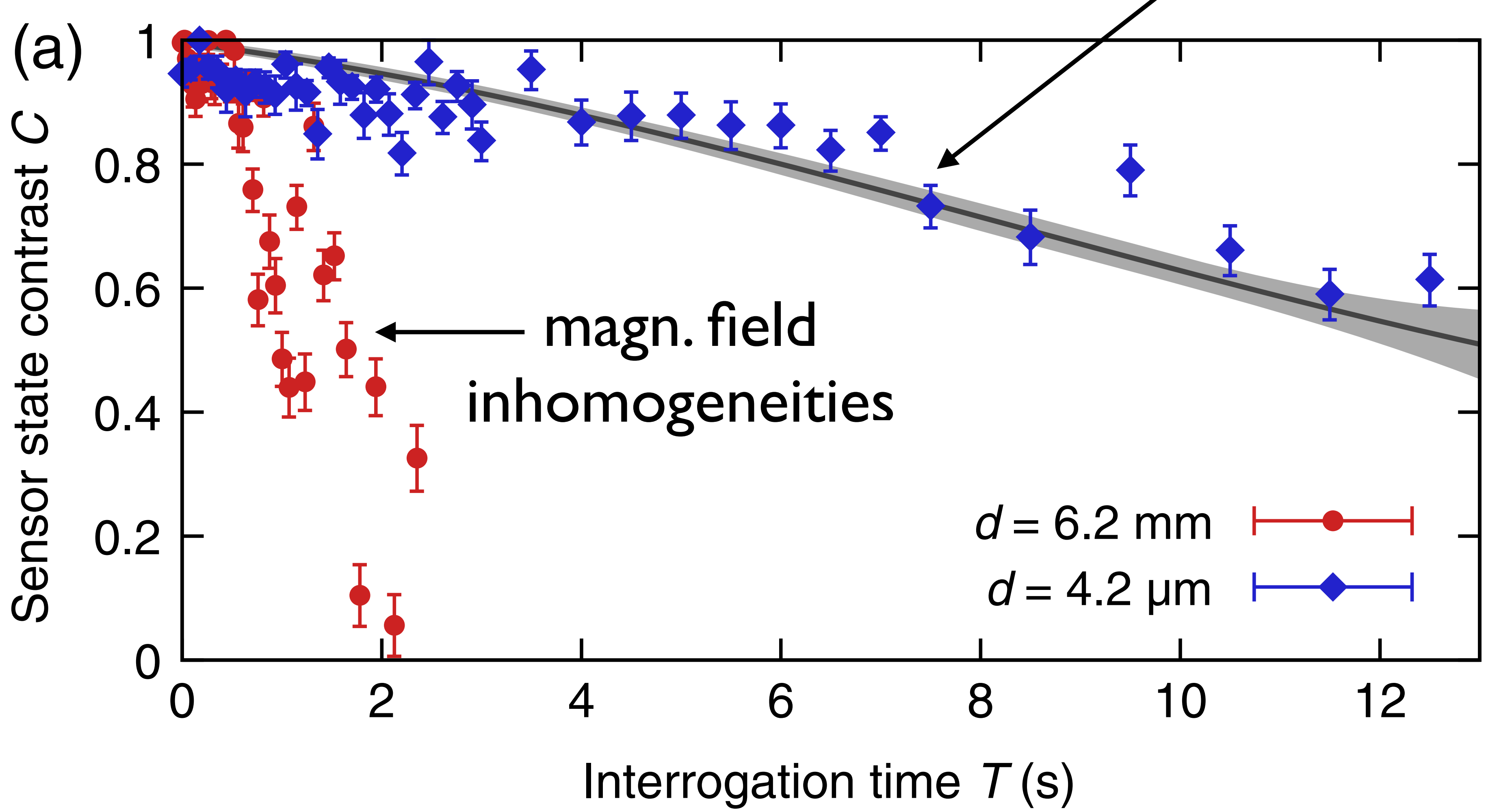
$$\rightarrow \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + e^{i\phi}|1\rangle|0\rangle)$$

T. Ruster, et al., Phys. Rev. X **7**, 031050 (2017)

Harnessing spatially correlated noise

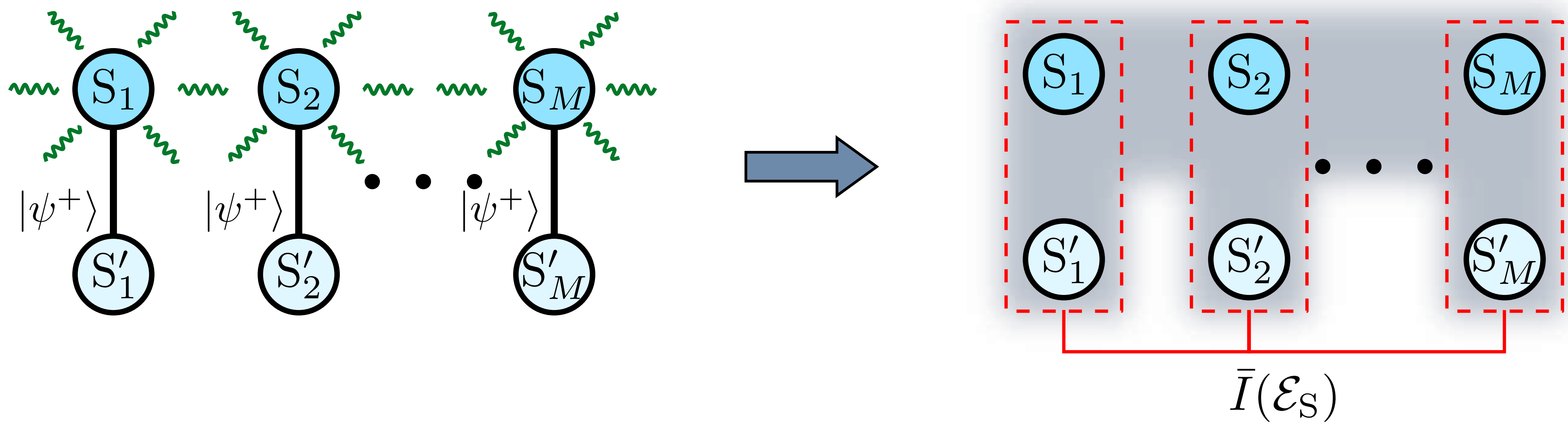
Entanglement-based magnetometry

$$H_G(t) = \frac{1}{2} B(t) \sum_k Z_k$$



Correlated Dynamics in Multipartite Setting

Several parties



$$\bar{I}(\mathcal{E}_\mathrm{S}) := \frac{1}{2M \log d} \left\{ \left[\sum_{i=1}^{M} S(\rho_\mathrm{S}^\mathrm{CJ}|_{\mathrm{S}_i\mathrm{S}'_i}) \right] - S(\rho_\mathrm{S}^\mathrm{CJ}) \right\}$$

with $\rho_\mathrm{S}^\mathrm{CJ}|_{\mathrm{S}_i\mathrm{S}'_i} = \mathrm{Tr}_{\{\forall \mathrm{S}_{j\neq i}\mathrm{S}'_{j\neq i}\}}(\rho_\mathrm{S}^\mathrm{CJ})$

Lower Bound Estimation

The exact experimental determination of I becomes impractical as the number of systems increases

Lower estimators

$$\bar{I}(\mathcal{E}) \geq \frac{1}{4M \ln d} \frac{C^2_{\mathcal{E}(\rho)}(X_1, \ldots, X_M)}{\|X_1\|^2 \ldots \|X_M\|^2}, \qquad \rho = \bigotimes_{i=1}^{M} \rho_i$$

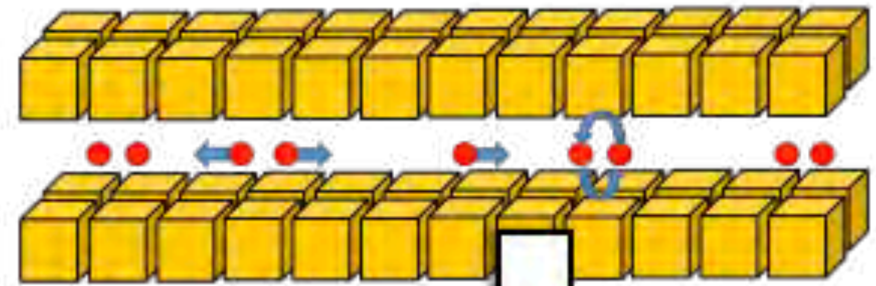
Operator norm

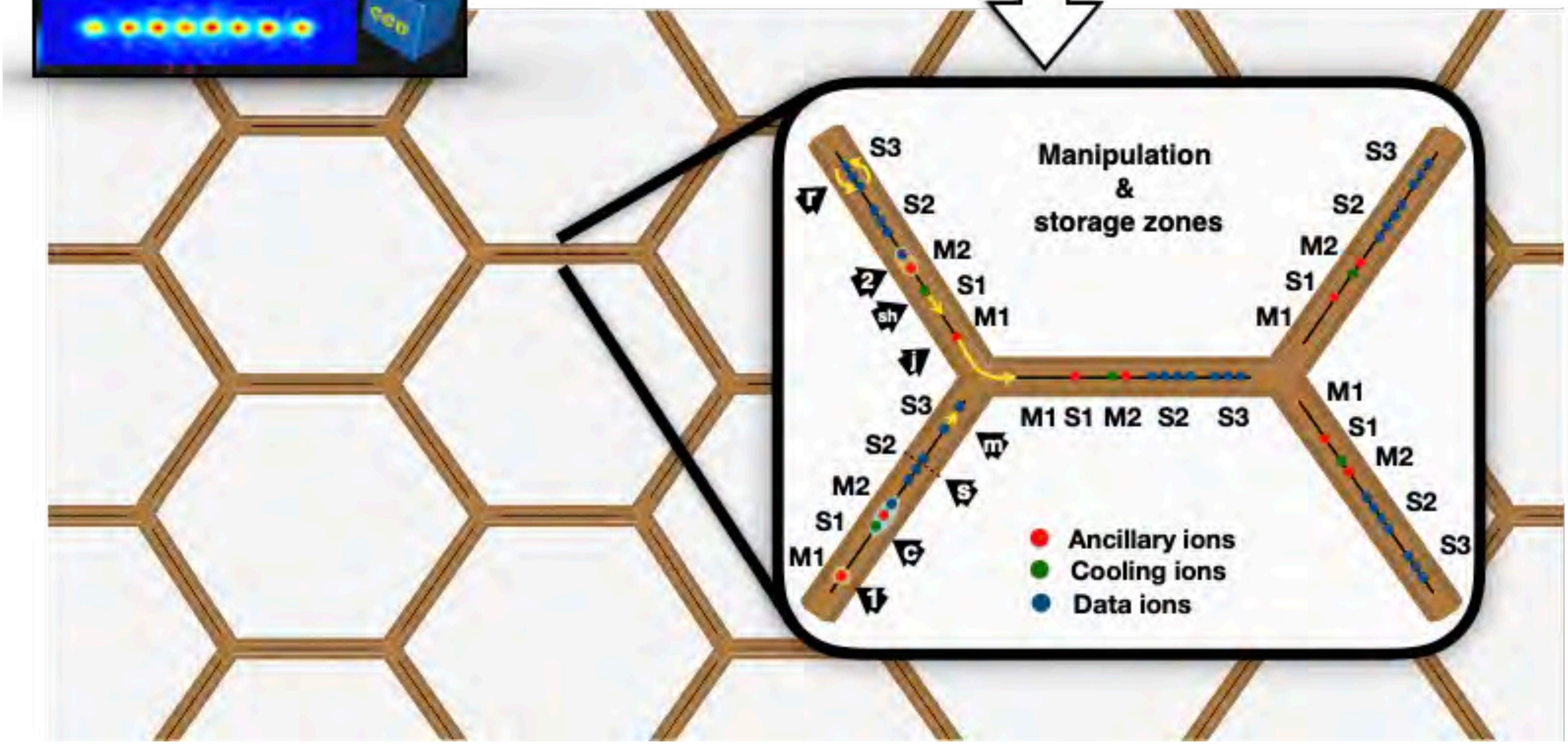
$$C^2_{\mathcal{E}(\rho)}(X_1, \ldots, X_M) = \langle X_1 \ldots X_M \rangle_{\mathcal{E}(\rho)} - \langle X_1 \rangle_{\mathcal{E}(\rho)} \ldots \langle X_M \rangle_{\mathcal{E}(\rho)}$$

Vision of scalable quantum computers

Linear Paul trap

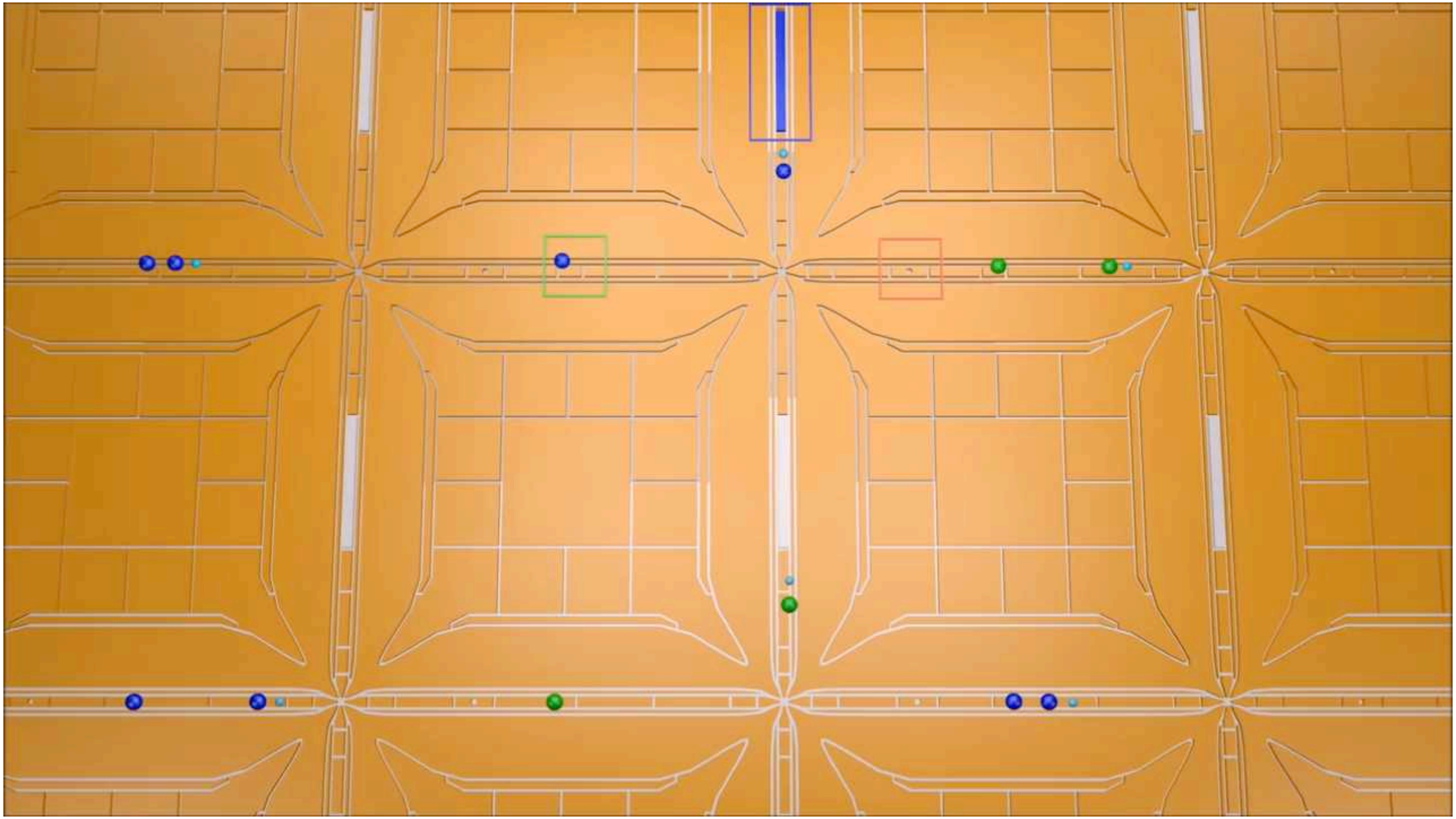
Segmented ion traps





Couple linear traps to build
larger 2D trap arrays as scalable qubit registers

Vision of scalable quantum computers



Understanding and mitigating correlated noise,
and quantum error correction will be crucial

W. Hensinger group
University of Sussex

Thanks!

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FAST TRACK COMMUNICATION

Quantifying spatial correlations of general quantum dynamics

Ángel Rivas and Markus Müller
Departamento de Física Teórica I, Universidad Complutense, E-28040 Madrid, Spain

Quantum
the open journal for quantum science

Experimental quantification of spatial correlations in quantum dynamics

Lukas Postler[1], Ángel Rivas[2,3], Philipp Schindler[1], Alexander Erhard[1], Roman Stricker[1], Daniel Nigg[1], Thomas Monz[1], Rainer Blatt[1,4], and Markus Müller[5]

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