Correlated Electron Dynamics and Nonequilibrium Dynamical Mean-Field Theory

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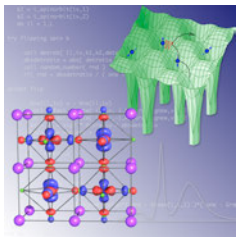


FOR 1346

Autumn School on Correlated Electrons:
DMFT at 25: Infinite Dimensions
Forschungszentrum Jülich, September 15-19, 2014

Outline

1. Quantum many-body systems in nonequilibrium
2. Nonequilibrium Green functions
3. Nonequilibrium Dynamical Mean-Field Theory
4. Interaction quench in the Hubbard model

Review on Nonequilibrium DMFT:

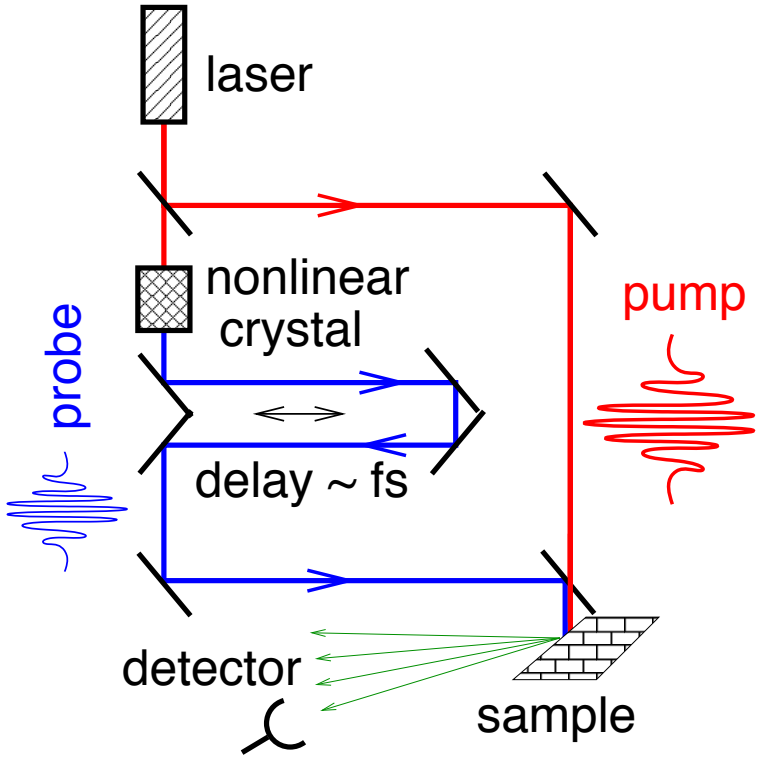
H. Aoki, N. Tsuji, M. Eckstein, M. Kollar, T. Oka, and P. Werner, Rev. Mod. Phys. **86**, 779 (2014)

1.
Quantum many-body systems in nonequilibrium

How to put
a quantum many-body system
out of equilibrium
and observe its relaxation

Time-resolved pump-probe spectroscopy

- Pump-probe setup:
  - Pump laser pulse:
    puts system into nonequilibrium
  - Probe laser pulse:
    looks at system after delay time



- Various time-resolved probes:
  - t.-r. ARPES: photoemitted **electrons**
  - t.-r. optical spectroscopy: transmitted/reflected **light**
  - t.-r. X-ray or electron **diffraction**: snapshots of atomic positions

Melting of a Charge Density Wave in $TbTe_3$

Schmitt, Kirchmann, Bovensiepen, Moore, Rettig, Krenz, Chu, Ru, Perfetti, Lu, Wolf, Fisher, Shen, Science '08

- trARPES on $TbTe_3$: 1.5-eV 50-fs pump pulse, 6-eV 90-fs probe pulse
- photodoping ➔ closing of CDW gap
  ➔ electron thermalization ➔ vibrational excitation

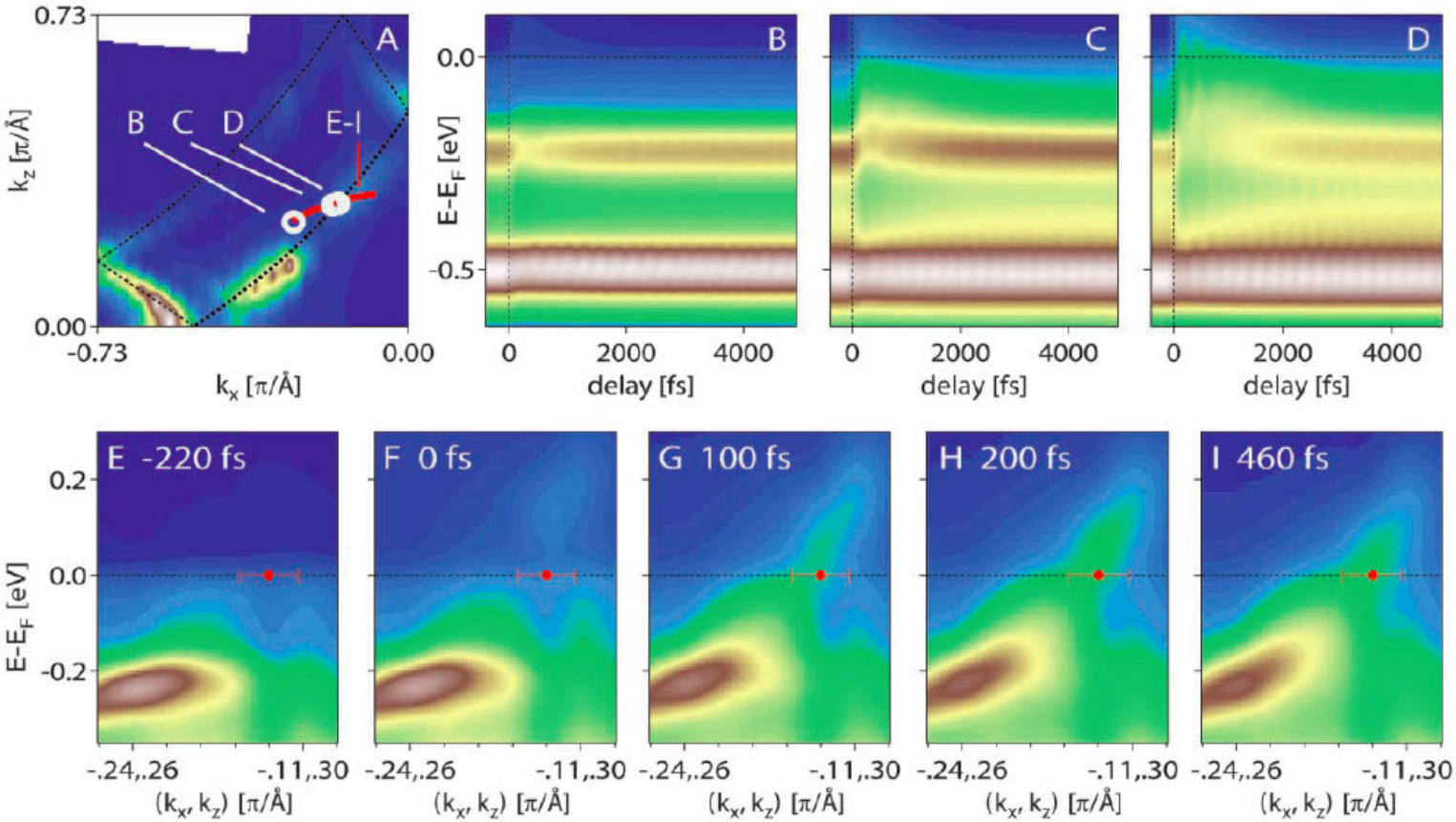


Fig. 3. (**A**) Detail of the FS plot in Fig. 1A′ with indicated positions (white circles) of time-resolved data shown in (**B**) to (**D**) for fixed k as a function of time delay. Indicated cut position (red line) of photoelectron intensity is shown as a function of energy, and position [(**E**) to (**I**)] for a momentum scan is shown as a function of time delays. All data were collected at 100 K and $F = 2$ mJ/cm^2. k_F is marked in (E) to (I) (red dot). Error bars indicate the distance to the neighboring sample points, which is a good estimate for the error of k_F.

Quenched Bose condensate

Abrupt increase of interaction of ^{87}Rb atoms:

Greiner, Mandel, Hänsch, Bloch '02

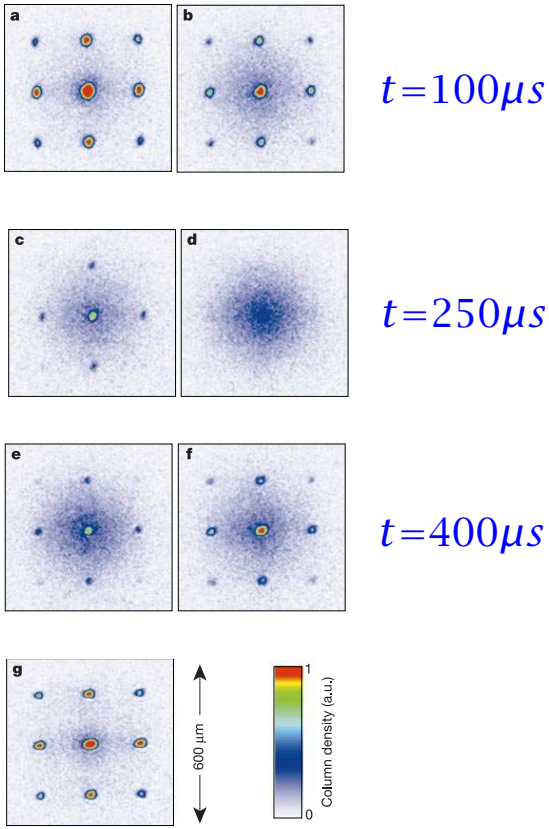
$\langle \hat{b}_k^\dagger \hat{b}_k \rangle$

$t=0\mu s$ $t=100\mu s$

$t=150\mu s$ $t=250\mu s$

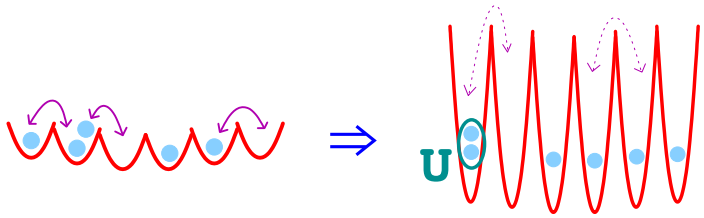
$t=350\mu s$ $t=400\mu s$

$t=550\mu s$



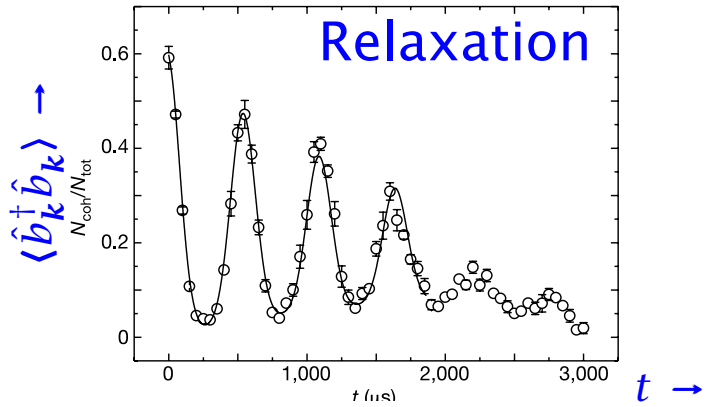
'collapse and revival'

$|\psi(0)\rangle$ = Bose condensate



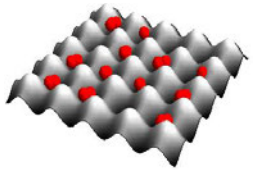
$$H \approx U \sum_i \hat{n}_i^2$$

$|\psi(t)\rangle = e^{-i\hat{H}t} |\psi(0)\rangle$ oscillates



Time scales in nonequilibrium dynamics

- **Time scales in pump-probe experiments**
  - Excitation due to pump pulse ~ 10...100 fs
  - Relaxation due to electron scattering ~ 0...1000 fs
  - Energy transfer to ion lattice ~ 1...10 ps
- **Time scales in cold-atom experiments**
  - Switching times ~ 1...1000 ms
  - Relaxation times ~ 1...1000 ms



➔ study relaxation of **isolated** quantum systems first

How can an isolated system relax to an equilibrium state?

Time evolution of isolated systems

- Schrödinger equation: $i\hbar \frac{d}{dt} |\psi(t)\rangle = H(t) \, |\psi(t)\rangle$
- Quantum quench: Prepare $|\psi_0\rangle$ and switch to H at $t = 0$
  - Time evolution for $t \geq 0$: $|\psi(t)\rangle = e^{-iHt} \, |\psi_0\rangle$
  - Energy after quench: $E = \langle\psi(t)|H|\psi(t)\rangle = \langle\psi_0|H|\psi_0\rangle$
  - Expectation values: $\langle A\rangle_t = \langle\psi(t)|A|\psi(t)\rangle$
- Thermalization:

$$\langle A\rangle_t \overset{t \to \infty}{\longrightarrow} \langle A\rangle_{\text{therm}} = \frac{\text{Tr}\, A\, e^{-\beta H}}{\text{Tr}\, e^{-\beta H}} \; ?$$

with effective β from $\langle H\rangle_{\text{therm}} = E$

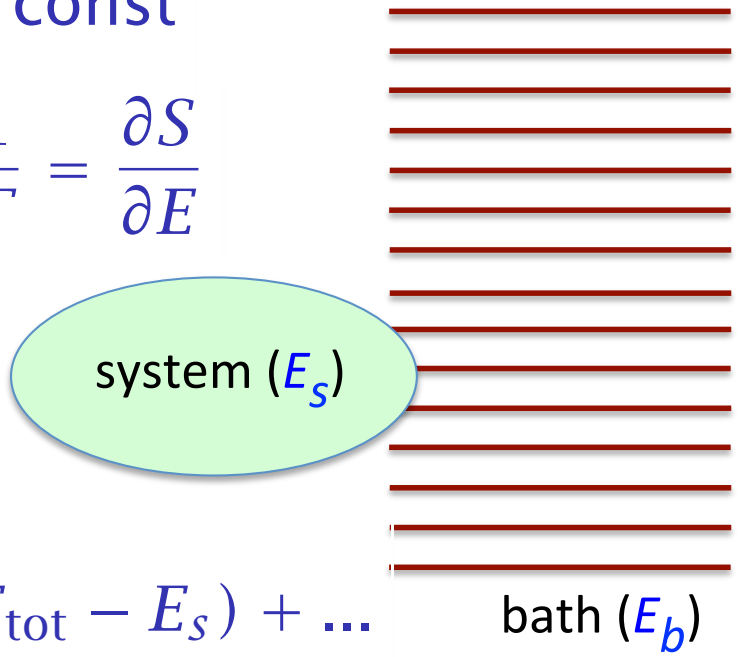
The thermal state:

Putting a system into equilibrium by coupling it to a *heat bath*

Gibbs ensemble for system + bath

Maxwell 1866, Boltzmann 1872, Gibbs 1878

- System + heat bath: $E_{\text{tot}} = E_s + E_b = \text{const}$
- Boltzmann relation: $S = \ln \Omega, \quad \beta = \frac{1}{T} = \frac{\partial S}{\partial E}$



- Obtain # of system states from bath:

$$\ln \Omega_b(E_{\text{tot}} - E_s) = \ln \Omega_b(E_{\text{tot}}) + \beta(E_{\text{tot}} - E_s) + \ldots$$

$$P(E_s) \propto \Omega_b(E_{\text{tot}} - E_s) \propto \exp(-\beta E_s)$$

- System in thermal state when in equilibrium with bath
- Microcanonical ensemble gives same results (in thermodyn. limit)

An equivalent *equilibrium* formulation:

Ensembles **containing microstates with same a priori probabilities**

Reformulation with fundamental postulate

Prediction for equilibrium state:

- *Fundamental postulate*:

  All accessible states equally probable $\Leftrightarrow S = -\mathrm{Tr}[\rho \ln \rho] = \max$

- A_i conserved $\Rightarrow$ fix $\mathrm{Tr}[\rho_{\text{ensemble}} A_i] = \langle A_i \rangle_{t=0}$

  $\Rightarrow \rho_{\text{ensemble}} \propto \exp(-\sum_i \lambda_i A_i)$ Boltzmann-Gibbs ensemble

von Neumann 1927, Jaynes 1957, ... Balian 1991

Reformulation with fundamental postulate

Prediction for equilibrium state for an isolated system

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Integrable systems: $H_{\mathrm{eff}} = \sum_{\alpha=1}^{L} \epsilon_\alpha n_\alpha$ $\Rightarrow$ many constants of motion

Jaynes '57
Girardeau '69
Rigol et al. '06
Cazalilla '06
Rigol et al.'07

- *Generalized Gibbs ensembles*: $\rho_{\mathrm{GGE}} \propto \exp(-\sum_\alpha \lambda_\alpha n_\alpha)$
- $\langle A \rangle_{t \to \infty} = \langle A \rangle_{\mathrm{GGE}}$ for simple observables and initial states

Kollar & Eckstein '08
Barthel & Schollwöck '08

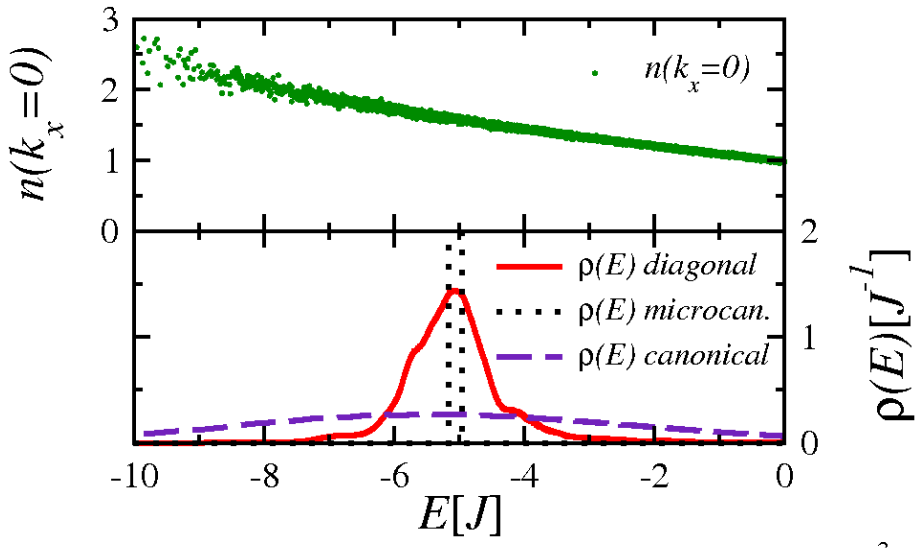
Why does a many-body system relax to the *thermal* state?

Eigenstate Thermalization Hypothesis (ETH)

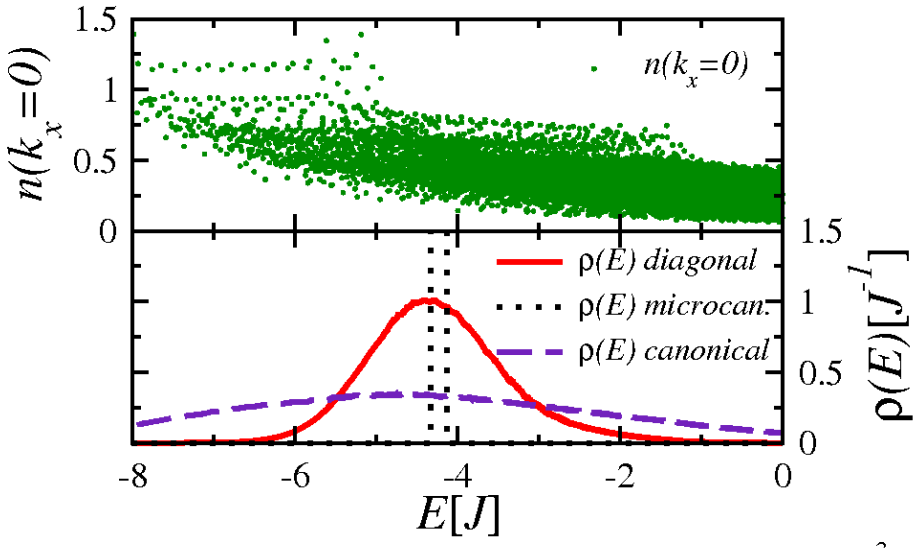
$\langle n|A|n\rangle \approx \mathcal{A}(E_n)$ + smaller, n-dep. terms

Deutsch PRA '91, Srednicki PRE '94
Rigol, Dunjko, Olshanii, Nature '08

Nonintegrable



Integrable



Eigenstate Thermalization Hypothesis (ETH)

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Deutsch PRA '91, Srednicki PRE '94
Rigol, Dunjko, Olshanii, Nature '08

- Energy dependence is that of microcanonical ensemble:

$$A_{\text{mic}}(E) = \text{Tr}[\rho_{\text{mic}}(E_n)\, A]$$

$$= \frac{1}{Z_{\text{mic}}} \sum_{E-\delta E < E_n < E} \langle n|A|n\rangle = \mathcal{A}(E) + \text{smaller terms}$$

- Long-time average tends to thermal value:

$$\overline{A(t)} = \overline{\langle\psi(t)|A|\psi(t)\rangle}$$

$$= \sum_n \underbrace{\langle n|A|n\rangle}_{\approx A_{\text{mic}}(E_n)} \underbrace{|\langle\psi(0)|n\rangle|^2}_{\text{peaked at } E} \approx A_{\text{mic}}(E)$$

- ETH sufficient for thermalization!

- **Nonequilibrium:**

  Thermalization is due to dependence of expectation values *only on energy*

- **Equilibrium:**

  Thermal Gibbs state is due to immersion in *structureless heat bath*

2.
Nonequilibrium Green functions

Quantum time evolution

- Hamiltonian: $\mathcal{H}(t) = H(t) - \mu N(t)$
- Density matrix: $\rho(0) = \frac{1}{Z} e^{-\beta \mathcal{H}(0)} = \frac{1}{Z} \sum_n e^{-\beta E_n} |n\rangle\langle n|$
- Propagator: $\rho(t) = U(t,0)\,\rho(0)\,U(0,t)$

$$\frac{d}{dt} U(t,t') = -i\mathcal{H}(t) U(t,t')$$

$$U(t,t') = \begin{cases} \mathrm{T} \exp\left(-i \int_{t'}^{t} d\bar{t}\, \mathcal{H}(\bar{t})\right) & \text{for } t > t' \\ \bar{\mathrm{T}} \exp\left(-i \int_{t'}^{t} d\bar{t}\, \mathcal{H}(\bar{t})\right) & \text{for } t < t' \end{cases}$$

- Expectation value of observable A:

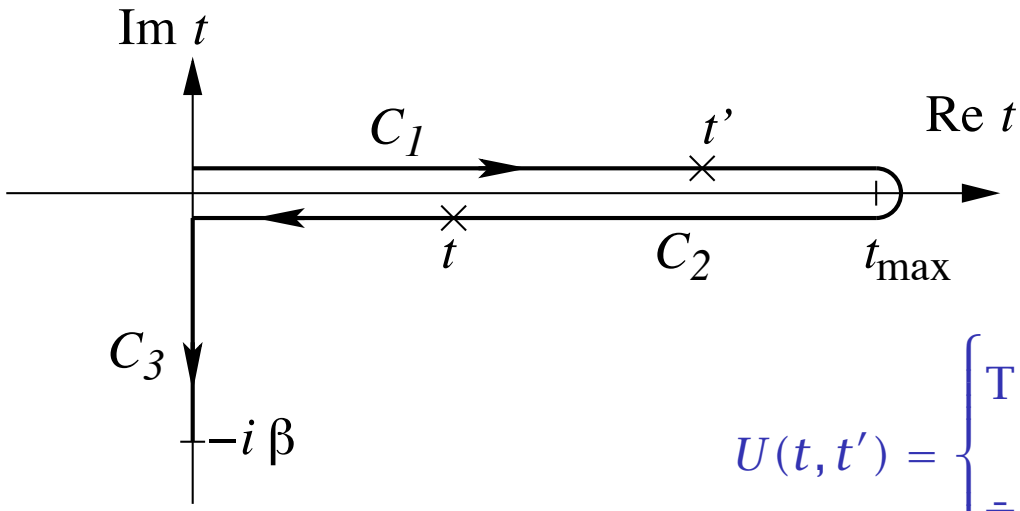
$$\langle A \rangle_t = \mathrm{Tr}\Big[\rho(t) A\Big] = \frac{1}{Z} \mathrm{Tr}\Big[U(-i\beta,0) U(0,t)\, A\, U(t,0)\Big]$$

Kadanoff-Baym formalism with time contour

- Expectation value of observable A:

$$\langle A \rangle_t = \frac{1}{Z} \mathrm{Tr}[U(-i\beta, 0) U(0, t)\, A\, U(t, 0)]$$

- Represent as integral over time contour $C_1 + C_2 + C_3$:



$$U(t, t') = \begin{cases} \mathrm{T} \exp\left(-i \int_{t'}^{t} d\bar{t}\, \mathcal{H}(\bar{t}) \right) & \text{for } t > t' \\ \bar{\mathrm{T}} \exp\left(-i \int_{t'}^{t} d\bar{t}\, \mathcal{H}(\bar{t}) \right) & \text{for } t < t' \end{cases}$$

- Insert formal time dependence into Schrödinger operator A:

$$\langle A \rangle_t = \frac{\mathrm{Tr}\, \mathrm{T}_C\, A(t)\, \exp[-i \int_C d\bar{t}\, \mathcal{H}(\bar{t})]}{\mathrm{Tr}\, \mathrm{T}_C\, \exp[-i \int_C d\bar{t}\, \mathcal{H}(\bar{t})]}$$

Contour calculus

$$g(t \in C) = \begin{cases} g^+(t) & \text{if } t \in [0, t_{\max}] \text{ on } C_1, \\ g^-(t) & \text{if } t \in [0, t_{\max}] \text{ on } C_2, \\ g^|(-i\tau) & \text{if } t = -i\tau \text{ on } C_3,\ \tau \in [0, \beta], \end{cases}$$

$$\int_C dt\, g(t) = \int_0^{t_{\max}} dt\, g^+(t) - \int_0^{t_{\max}} dt\, g^-(t) - i \int_0^{\beta} d\tau\, g^|(-i\tau)\,,$$

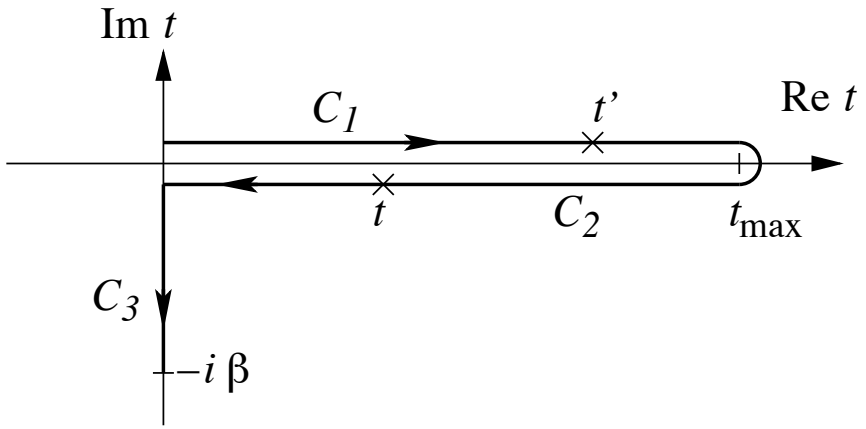
$$[a * b](t, t') = \int_C d\bar{t}\, a(t, \bar{t}) b(\bar{t}, t')\,,$$

$$\partial_t g(t) = \begin{cases} \partial_t g(t^\pm) & t \in C_{1,2} \\ i\partial_\tau g(-i\tau) & t = -i\tau \in C_3 \end{cases},$$

$$\theta_C(t, t') = \begin{cases} 1 & \text{for } t >_C t'\,, \\ 0 & \text{otherwise,} \end{cases}$$

$$\delta_C(t, t') = \partial_t \theta_C(t, t')\,,$$

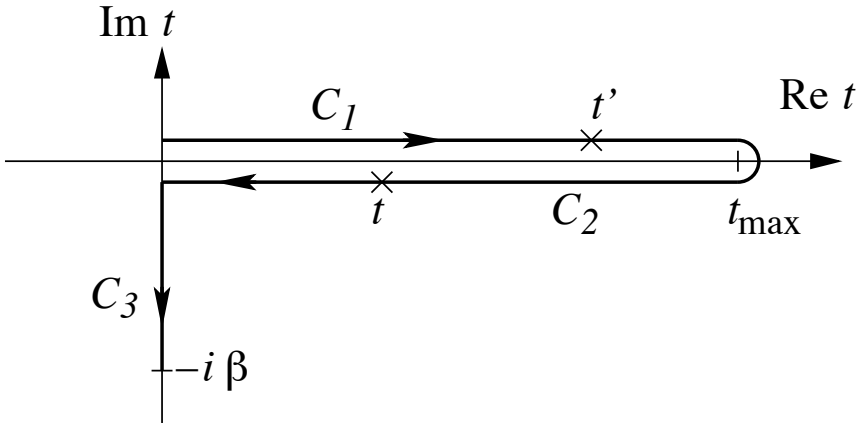
$$\int_C d\bar{t}\, \delta_C(t, \bar{t}) g(\bar{t}) = g(t)$$



Contour Green functions

- Contour time ordering:

$$\mathrm{T}_C\, A(t)B(t') = \begin{cases} AB & \text{if } t >_C t' \\ \pm BA & \text{if } t <_C t' \end{cases}$$



- Action: $S = -i \int_C dt\, \mathcal{H}(t)$

- Green function with 2 time arguments on branches C_1 or C_2 or C_3:

$$G(t,t') = -i\langle c(t)c^\dagger(t')\rangle = -\frac{i}{Z}\mathrm{Tr}\left[\mathrm{T}_C\left\{\exp(S)c(t)c^\dagger(t')\right\}\right]$$

- Let $G_{ab}(t,t')$ have time arguments on branches $a,b = 1,2,3$

- Symmetries: $G_{11}(t,t') = G_{12}(t,t')$ for $t \le t'$ etc.

$$G_{13}(t,\tau') = G_{23}(t,\tau'),$$

Keldysh Green functions

Retarded, Advanced, Keldysh, Mixed, Matsubara, lesser, greater GFs:

$$G^R(t,t') = \tfrac{1}{2}(G_{11} - G_{12} + G_{21} - G_{22}) = -i\theta(t-t')\langle\{c(t), c^\dagger(t')\}\rangle$$

$$G^A(t,t') = \tfrac{1}{2}(G_{11} + G_{12} - G_{21} - G_{22}) = i\theta(t'-t)\langle\{c(t), c^\dagger(t')\}\rangle$$

$$G^K(t,t') = \tfrac{1}{2}(G_{11} + G_{12} + G_{21} + G_{22}) = -i\langle[c(t), c^\dagger(t')]\rangle$$

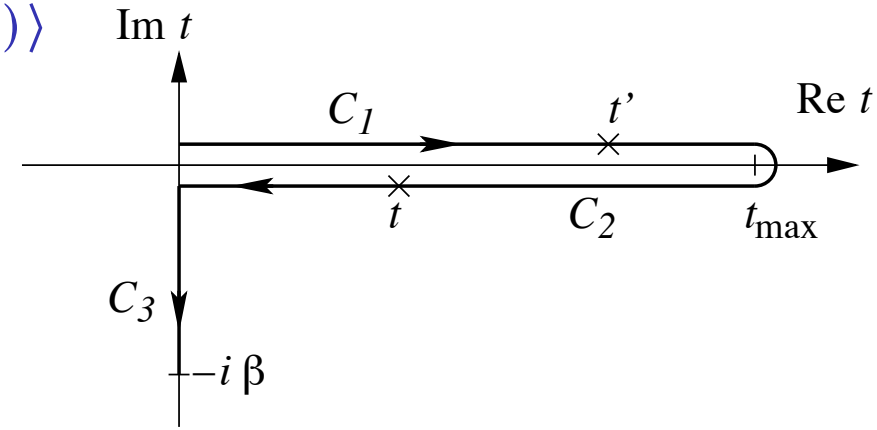
$$G^{\urcorner}(t,\tau') = \tfrac{1}{2}(G_{13} + G_{23}) = i\langle c^\dagger(\tau')c(t)\rangle$$

$$G^{\ulcorner}(\tau,t') = \tfrac{1}{2}(G_{31} + G_{32}) = -i\langle c(\tau)c^\dagger(t')\rangle$$

$$G^M(\tau,\tau') = -iG_{33} = -\langle \mathrm{T}_\tau\, c(\tau)c^\dagger(\tau')\rangle$$

$$G^<(t,t') = G_{12} = i\langle c^\dagger(t')c(t)\rangle$$

$$G^>(t,t') = G_{21} = -i\langle c(t)c^\dagger(t')\rangle$$



Noninteracting case and Self-energy

- Free electrons: $\mathcal{H}_0(t) = \sum_{\boldsymbol{k}} [\epsilon_{\boldsymbol{k}}(t) - \mu] c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}}$
- Contour GF: $G_{0,\boldsymbol{k}}(t,t') = -i \langle c_{\boldsymbol{k}}(t) c^\dagger_{\boldsymbol{k}}(t') \rangle$
- EOM: $[i\,\partial_t + \mu - \epsilon_{\boldsymbol{k}}(t)] G_{0,\boldsymbol{k}}(t,t') = \delta_C(t,t')$
- Def. of inverse GF: $G^{-1}_{0,\boldsymbol{k}}(t,t') = [i\,\partial_t + \mu - \epsilon_{\boldsymbol{k}}(t)] \delta_C(t,t')$
- Solution: $G_{0,\boldsymbol{k}}(t,t') = -i \Big[\theta_C(t,t') - f(\epsilon_{\boldsymbol{k}}(0) - \mu) \Big] e^{-i \int_{t'}^{t} d\bar{t}\, [\epsilon_{\boldsymbol{k}}(\bar{t}) - \mu]}$

- Self-energy Σ: 1-particle irreducible amputated Feynman diagrams
- Dyson equation for full GF: $G = G_0 + G_0 * \Sigma * G$
- Def. of inverse of full GF: $G^{-1} = G_0^{-1} - \Sigma$

3.
Nonequilibrium Dynamical Mean-Field Theory

The DMFT philosophy

- Start from limit of infinite lattice dimension $d \to \infty$
- Scale the kinetic energy, i.e., NN hopping amplitude $t_{ij} \propto \frac{1}{\sqrt{d}}$
- Map lattice problem onto

  dynamic single-site problem with *self-consistency condition*

  [e.g. single-impurity Anderson model (SIAM)]

  and solve numerically

- Extend to e.g.
  - finite d using clusters, dual fermions, dyn. vertex approx., ...
  - magnetic phases, phonons, ...
  - input from density functional theory, ...

The cavity method I

- Time-dep. Hubbard model: $H(t) = \sum_{ij\sigma} t_{ij}(t)\, c^\dagger_{i\sigma} c_{j\sigma} + U(t) \sum_i n_{i\uparrow} n_{i\downarrow}$
- Pick out single site *i=0* from lattice action:

Equil.: Georges et al. RMP 1996
Noneq.: Gramsch et al PRB 2014

$$S = S_0 + \Delta S + S^{(0)},$$

$$S_0 = -i \int_C dt \left[U(t) n_{0\uparrow}(t) n_{0\downarrow}(t) - \mu \sum_\sigma n_{0\sigma}(t) \right],$$

$$\Delta S = -i \int_C dt \left[\sum_{i\neq 0,\sigma} t^\sigma_{i0}(t) c^\dagger_{i\sigma}(t) c_{0\sigma}(t) + \text{h.c.} \right],$$

$$S^{(0)} = -i \int_C dt\, \mathcal{H}^{(0)}(t)$$

- Integrate out rest of lattice: $S_{\text{eff}} = S_0 + \tilde{S}$

$$\begin{aligned} Z &= \text{Tr} \left[\text{T}_C \left\{ \exp(S_0 + \Delta S + S^{(0)}) \right\} \right] \\ &= \text{Tr}_0 \left[\text{T}_C \left\{ \exp(S_0) \text{Tr}_{\text{rest}} \left(\exp(\Delta S + S^{(0)}) \right) \right\} \right] \\ &= \text{Tr}_0 \left[\text{T}_C \left\{ \exp(S_0 + \tilde{S}) \right\} \right] Z_{S^{(0)}} \end{aligned}$$

The cavity method II

- Result of integration over lattice sites *i≠0*:

$$\tilde{S} = -i \sum_{n=1}^{\infty} \sum_{\sigma_1 \ldots \sigma_n'} \int_C dt_1 \ldots \int_C dt_n' \, \Lambda_{\sigma_1 \ldots \sigma_n'}(t_1, \ldots, t_n') \, c^{\dagger}_{0\sigma_1}(t_1) \ldots c_{0\sigma_n'}(t_n')$$

- Hybridization functions:

$$\Lambda_{\sigma_1 \ldots \sigma_n'}(t_1, \ldots, t_n') = \frac{(-i)^{n-1}}{n!^2} \sum_{i_1, \ldots, j_n} t_{0 i_1}(t_1) \cdots t_{j_n 0}(t_n') \, G^{(0),\mathrm{c}}_{i_1 \sigma_1, \ldots, j_n \sigma_n'}(t_1, \ldots, t_n')$$

cavity Green function (site *i=0* removed)

- Power counting for $d \to \infty$:

$$\Lambda_{\sigma_1 \ldots \sigma_n'}(t_1, \ldots, t_n') \propto \underbrace{\sum_{i_1, \ldots, j_n}}_{\propto d^{2n}} \underbrace{t_{0 i_1}(t_1) \ldots t_{j_n 0}(t_n')}_{\propto (\sqrt{d})^{-2n}} \underbrace{G^{(0),\mathrm{c}}_{(i_1 \sigma_1), \ldots, (j_n \sigma_n')}(t_1, \ldots, t_n')}_{\propto (\sqrt{d})^{-2(2n-1)}}$$

$$\propto \frac{1}{d^{n-1}}$$

- Only one-particle Green functions (*n=1*) remain in hybridization!

The DMFT action

- Action for the cavity site *i=0*:

$$S_{\text{eff}} = -i \int_C dt \left[U(t) n_\uparrow(t) n_\downarrow(t) - \mu \sum_\sigma n_\sigma(t) \right] - i \int_C dt_1 \int_C dt_2 \sum_\sigma \Lambda_\sigma(t_1, t_2) c_\sigma^\dagger(t_1) c_\sigma(t_2)$$

- Hybridization function:

$$\Lambda_\sigma(t, t') = \sum_{i,j} t_{0i}(t) G^{(0),c}_{ij\sigma}(t, t') t_{j0}(t')$$

- Self-consistency for NN hopping on the Bethe lattice $t_{ij} = \dfrac{v}{\sqrt{z}}$

$$\Lambda_\sigma(t, t') = v(t) G_\sigma(t, t') v(t')$$

Local self-energy and self-consistency

- Lattice and impurity Green function:

$$G_{ij}(t,t') = -i\langle c_i(t)c_j^\dagger(t')\rangle_S \qquad\qquad G(t,t') = G_{00}(t,t')$$

- Lattice and impurity self-energies:

$$(G_{\rm lat}^{-1})_{ij}(t,t') = [\delta_{ij}(i\partial_t+\mu) - t_{ij}(t)]\delta_C(t,t') - (\Sigma_{\rm lat})_{ij}(t,t')$$

$$G^{-1}(t,t') = (i\partial_t+\mu)\delta_C(t,t') - \Lambda(t,t') - \Sigma(t,t')$$

- For DMFT action: $(\Sigma_{\rm lat})_{ij}(t,t') = \delta_{ij}\Sigma(t,t')$ local self-energy!

- Self-consistency conditions: lattice and impurity Dyson equation

$$\int_C dt_1 \sum_l \Big[[\delta_{il}(i\partial_t+\mu) - t_{il}(t)]\delta_C(t,t_1) - \Sigma(t,t_1)\Big]G_{lj}(t_1,t') = \delta_{ij}\delta_C(t,t')$$

$$\int_C dt_1\, [(i\partial_t+\mu)\delta_C(t,t_1) - \Lambda(t,t_1) - \Sigma(t,t_1)]G(t_1,t') = \delta_C(t,t')$$

Solution of DMFT equations by iteration

- DMFT iteration:
  - start from a hybridization function Λ
  - obtain impurity Green function $G(t,t') = -i\langle c(t)c^\dagger(t')\rangle$
  - obtain self-energy Σ from impurity Dyson equation
  - obtain new local Green function $G(t,t')$ from lattice Dyson eq.
  - obtain new hybridization function Λ

- Must solve Volterra-type integro-differential eqs.
- Can be implemented as time-propagation scheme

see RMP 2014 & references

Real-time impurity solvers

- **Many-body perturbation theory**
  - Weak-coupling perturbation theory

    [sample code available as Supp.Mat. for RMP **86**, 779 (2014)]
  - Strong-coupling perturbation theory
- **Continuous-time Quantum Monte Carlo**
- **Hamiltonian-based methods / exact diagonalization**
- **Falicov-Kimball model**

see Lecture Notes for references

4.
Interaction quench in the Hubbard model

$$H = \underbrace{\sum_{ij\sigma} t_{ij}\, c^{\dagger}_{i\sigma} c_{j\sigma}}_{= \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}}\, c^{\dagger}_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma}} + U(t) \sum_{i} n_{i\uparrow}\, n_{i\downarrow}$$

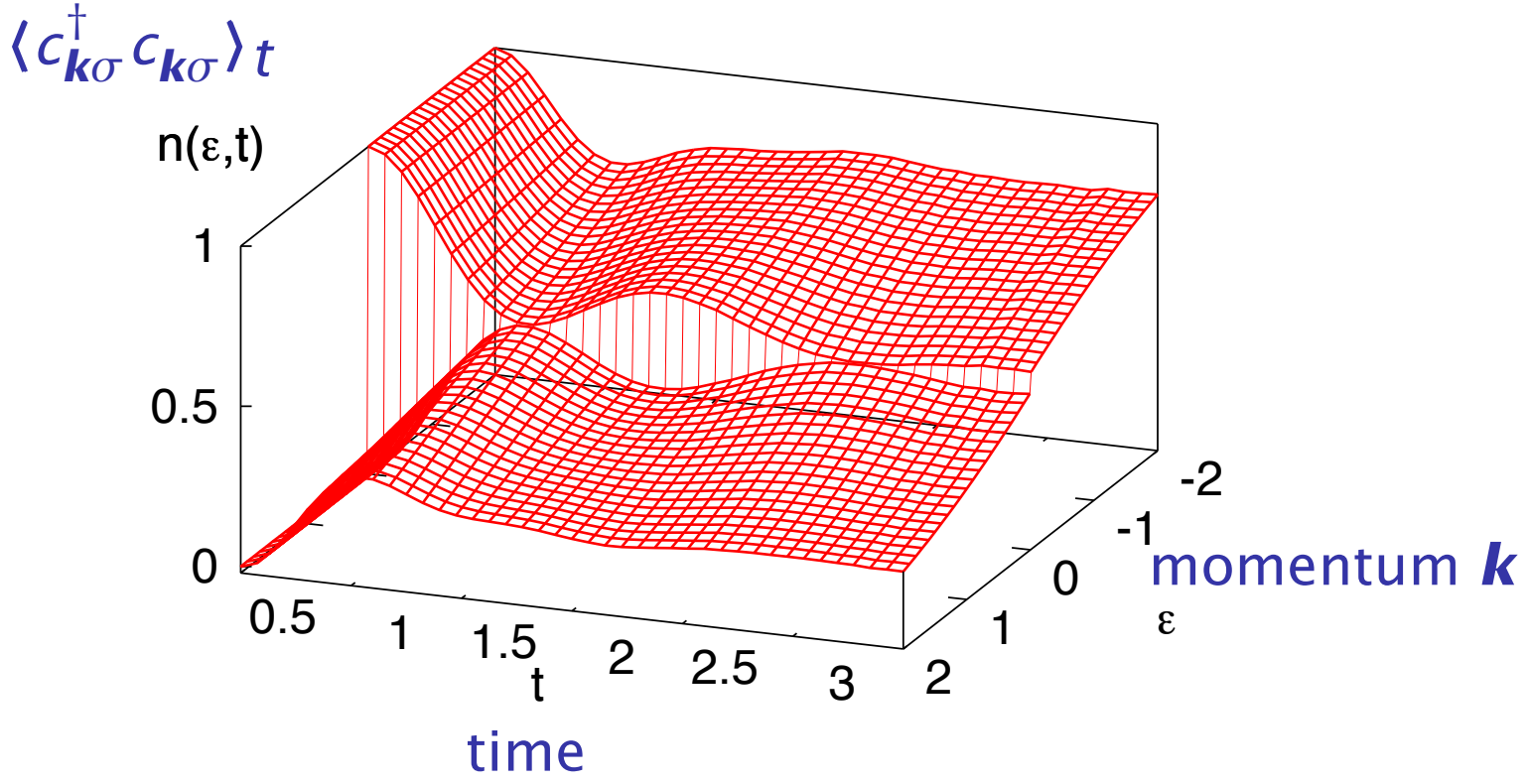
Strong-coupling regime:
collaps-and-revival oscillations

Hubbard interaction quench: Collapse & revival

Hubbard model in DMFT: (bandwidth = 4, density $n = 1$)

Eckstein et al., PRL '09, PRB '10 [CT-QMC]

Large interaction quench to $U = 5$



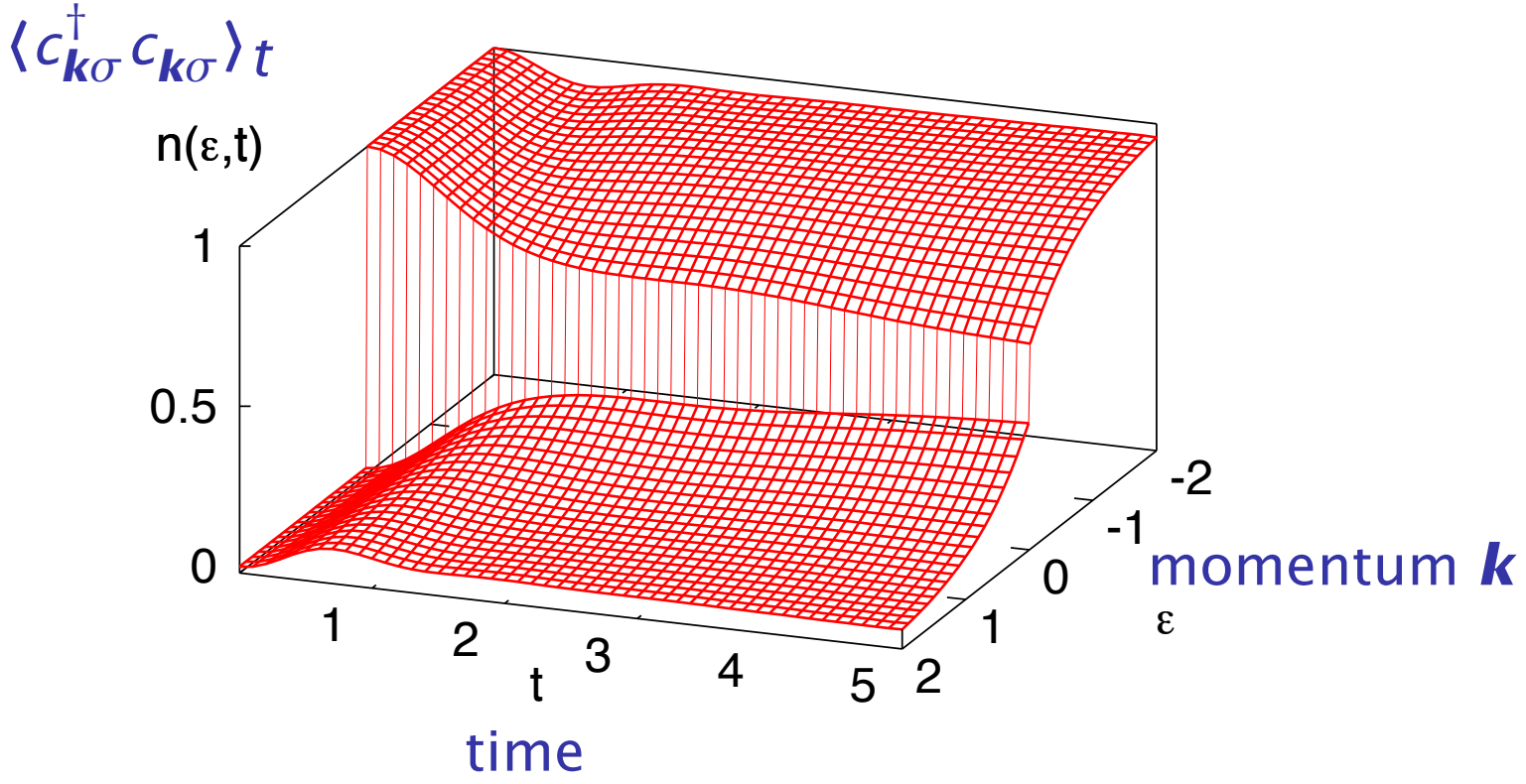
Collapse-and-revival oscillations
due to vicinity of atomic limit ($U = \infty$)

Weak-coupling regime:
metastable prethermalized state

Hubbard interaction quench: Prethermalization

Hubbard model in DMFT: (bandwidth $= 4$, density $n = 1$)

Small interaction quench to $U = 2$



Slow relaxation: *Prethermalization plateaus*

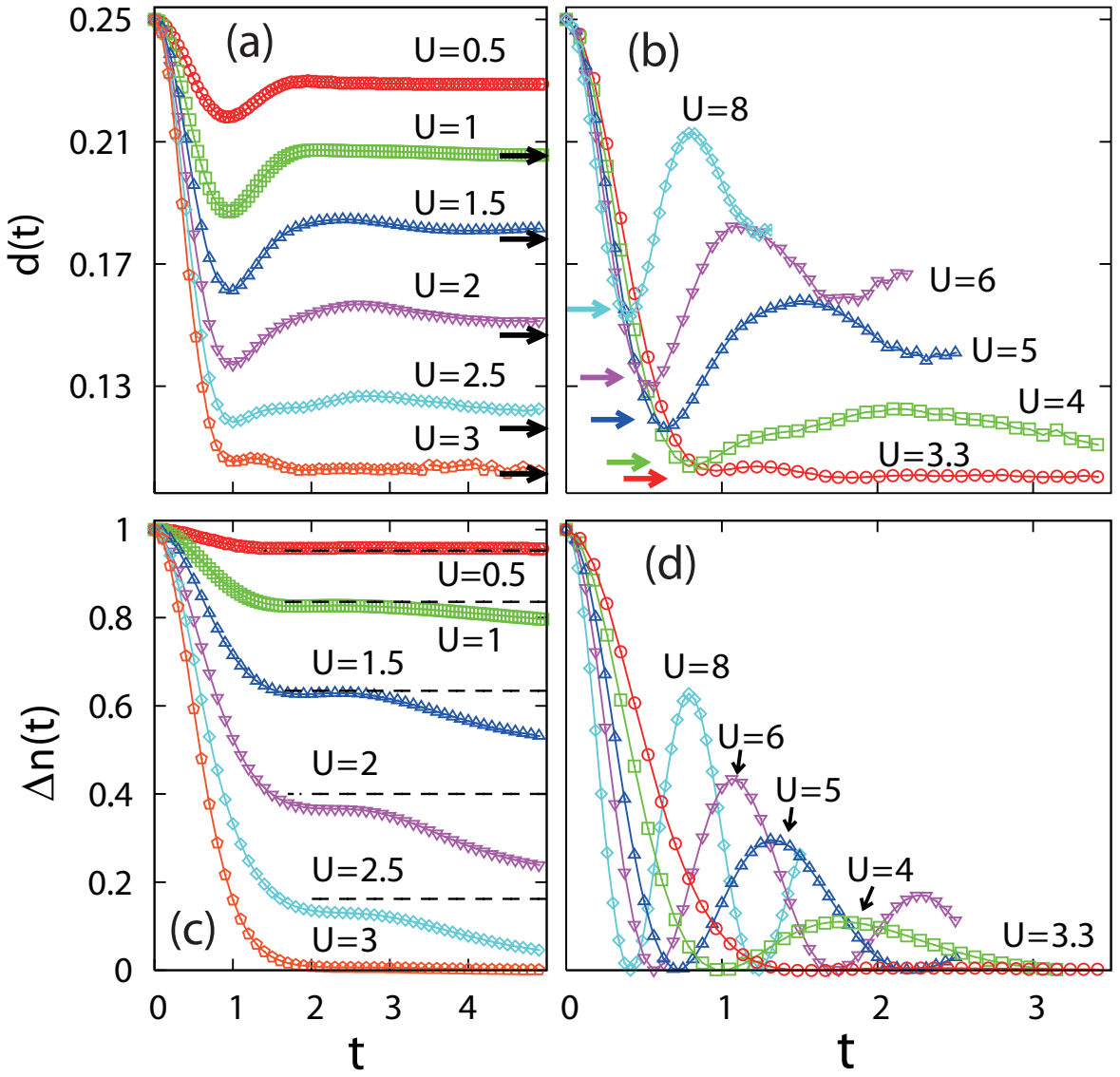
due to vicinity of free system ($U = 0$) Berges et al. PRL '04
Moeckel & Kehrein PRL '08

Hubbard interaction quench from 0 to U

Eckstein et al., PRL '09

Double occupation

Fermi surface discontinuity



Prethermalization regime

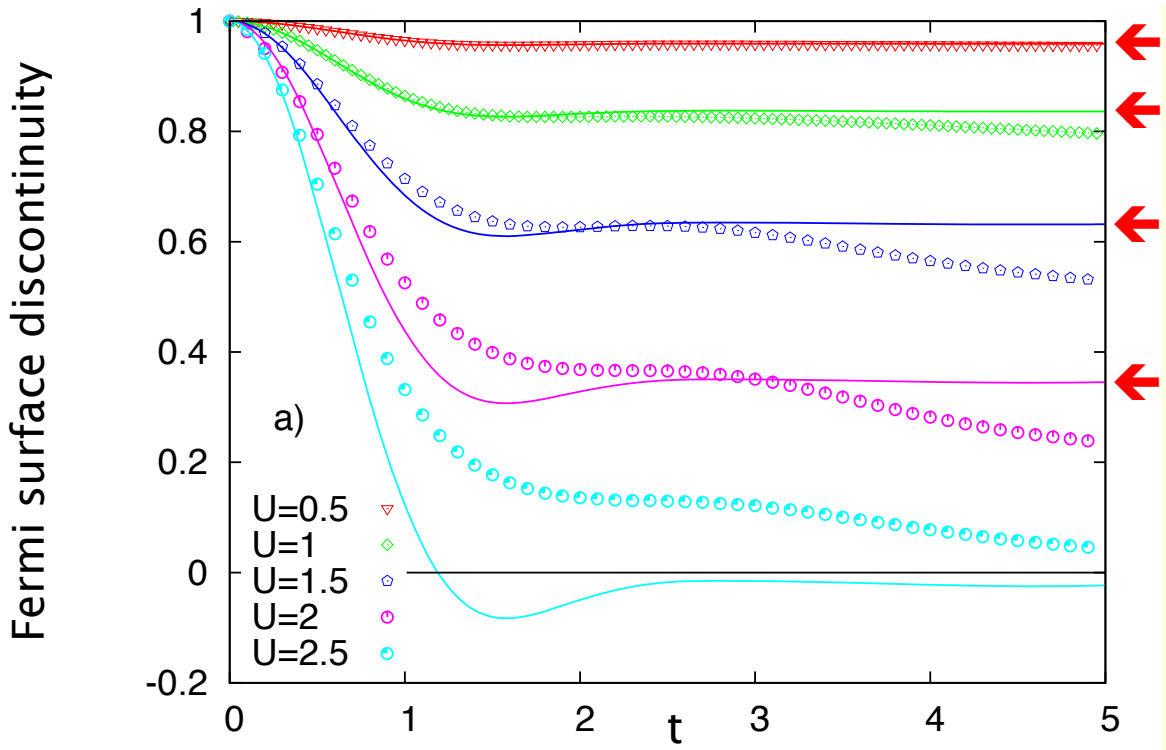
- Weak interactions: $H = H_0 + g\,H_1, \quad |g| \ll 1$

$$H_0 = \sum_\alpha \epsilon_\alpha\, c_\alpha^\dagger c_\alpha \qquad H_1 = \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta$$

- Unitary pert. theory: trafo, evolve, backtrafo

Moeckel & Kehrein PRL '08
Eckstein et al. PRB '10
Kollar et al. PRB '11

$$\langle A\rangle_t = \langle A\rangle_0 + g^2 F(Wt) + O(g^3) \overset{t\to\infty}{\longrightarrow} A_{\text{pretherm}} + \frac{g^2\cos(\Omega t+\phi)}{\text{const}\cdot(Wt)^\alpha}$$



Prethermalization plateau described by generalized Gibbs ensemble with approximate constants of motion

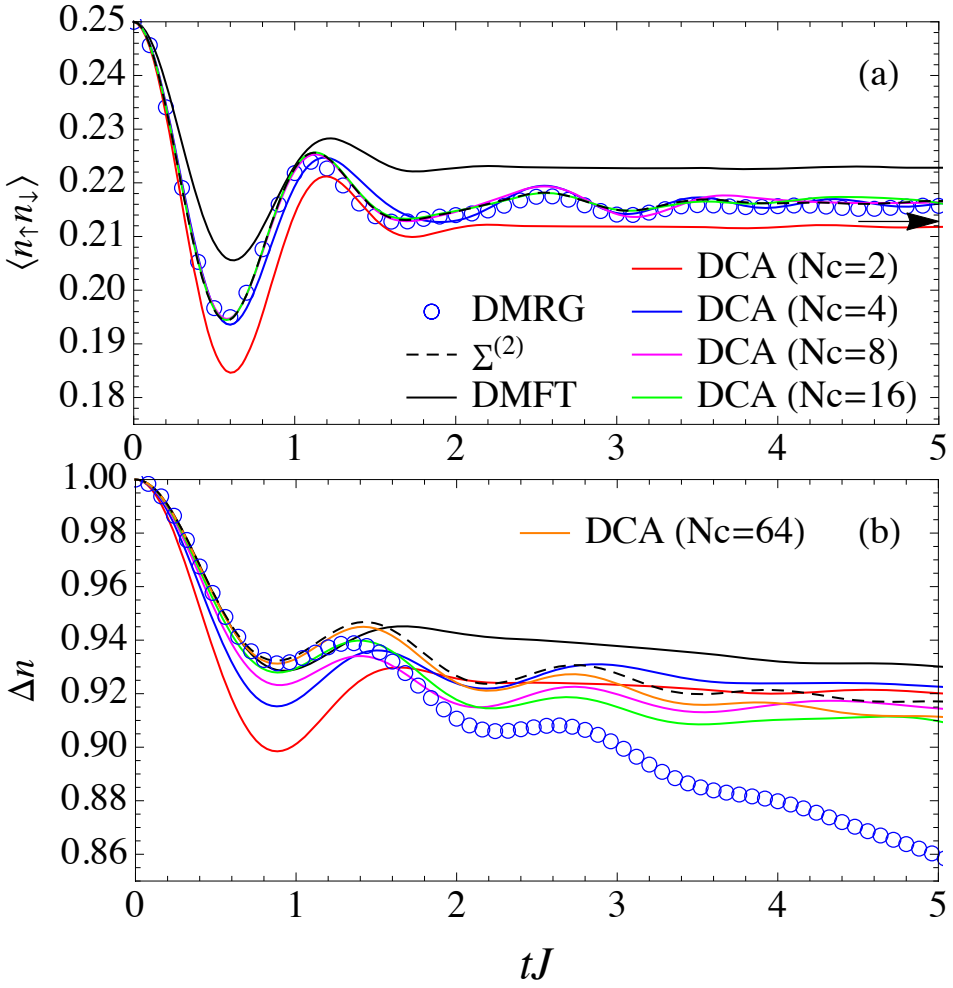
$$\rho_{\widetilde{\text{GGE}}} \propto \exp\left(-\sum_\alpha \lambda_\alpha \tilde{n}_\alpha\right)$$

Prethermalization in one and two dimensions

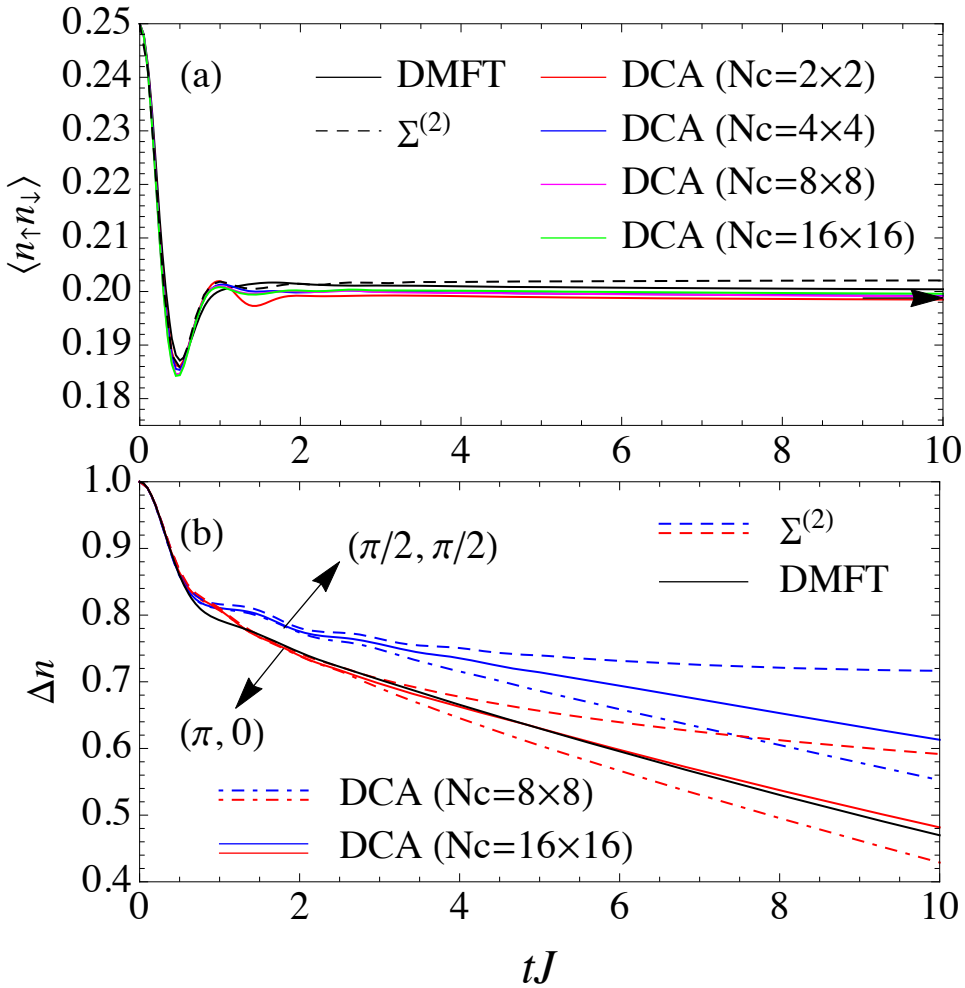
- Prethermalization less pronounced in low dimensions

Tsuji, Barmettler, Aoki, Werner, PRB '14

$d = 1$, U from 0 to 1



$d = 2$, U from 0 to 2

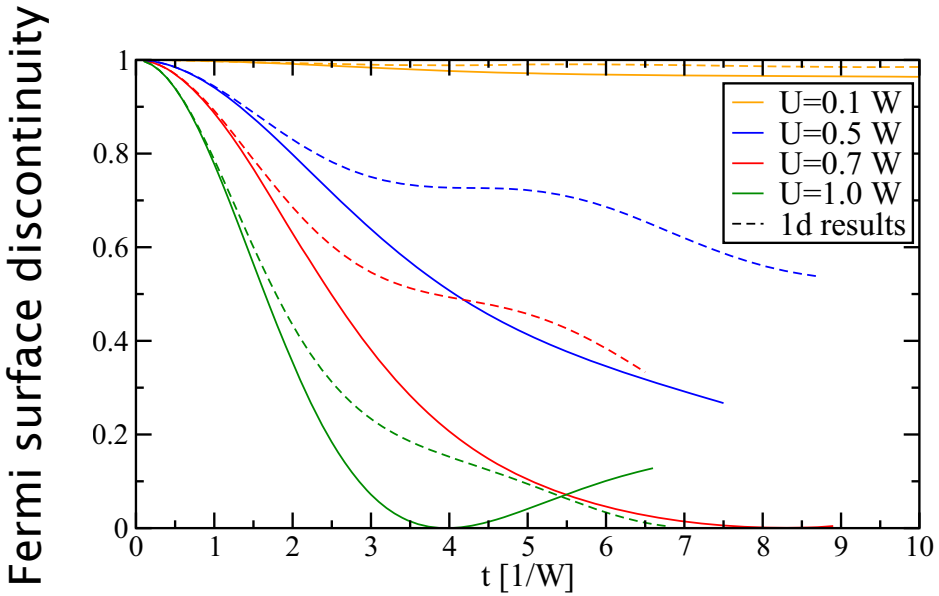


Prethermalization in one and two dimensions

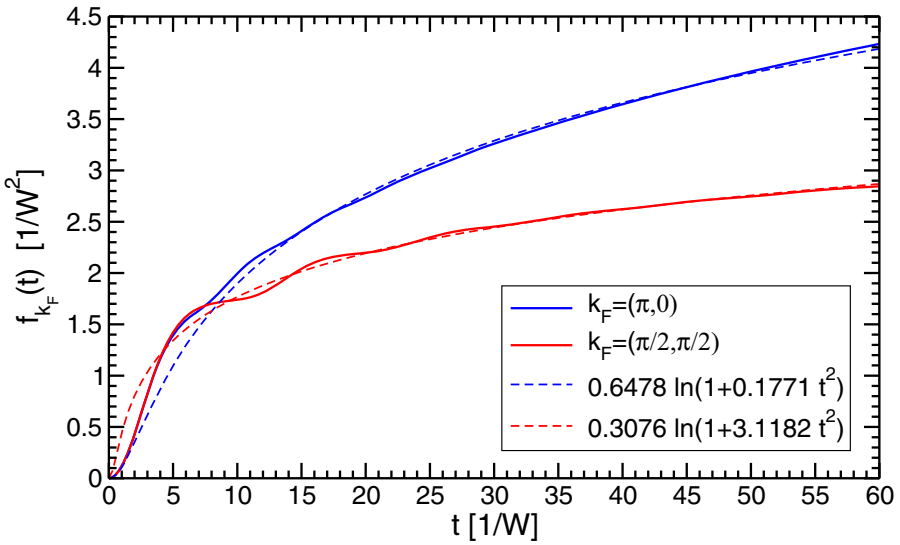
- Prethermalization less pronounced in low dimensions

short-time expansion, Hamerla and Uhrig, PRB '14

d = 1 and *d* = 2



d = 2



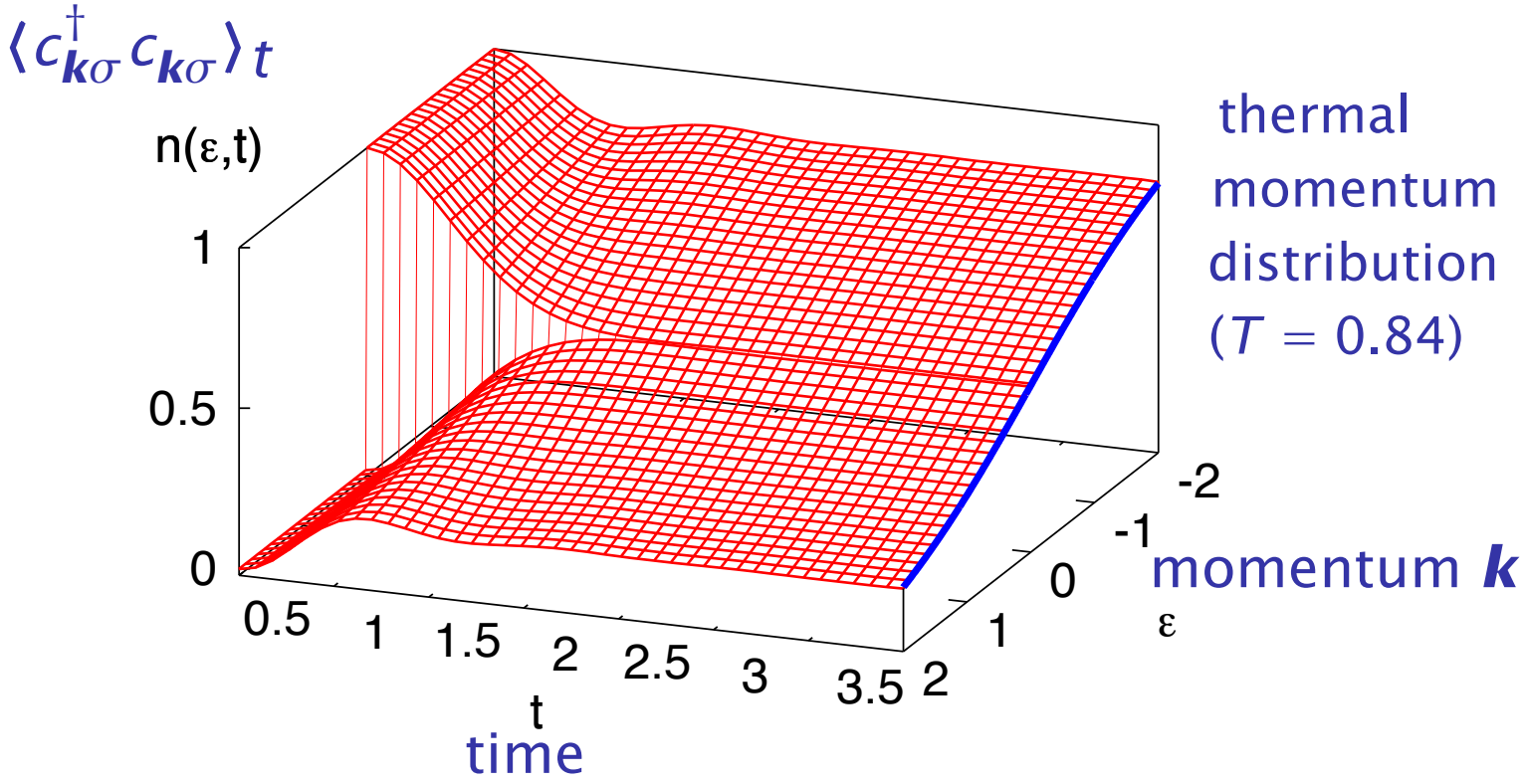
perturbative correction
does not reach a plateau

Intermediate-coupling regime:
fast thermalization

Hubbard interaction quench: Thermalization

Hubbard model in DMFT: (bandwidth = 4, density $n = 1$)

Intermediate interaction quench to $U = 3.3$



Fast thermalization at intermediate U

both prethermalization and oscillations disappear at $U_c^{dyn} \approx 3.2$

U_c^{dyn}~3.4 well-captured by time-dependent Gutzwiller approximation: Schiro & Fabrizio PRL 2010

Summary and Outlook

Summary and Outlook

- **Thermalization of correlated systems in nonequilibrium**

  - Eigenstate thermalization hypothesis:

    Thermalization is due to energy eigenstates that contribute only according to their energy

  - Thermalization does not occur in integrable systems

- **Nonequilibrium dynamical mean-field theory**

  - Controlled approximation for nonequilibrium problems

  - Many applications:
    quenches, pulses, periodic driving, ...
    cluster extensions, ...
    magnetic phases; phonons; bosons; ...