Exercise Sheet 11 due 23 January

1. *non-degenerate perturbation theory*

Consider an electron in a one-dimensional potential well of width L, with infinitely high barriers at $z{=}0$ and $z{=}L$. The potential energy inside the potential well is parabolic, of the form $V(z) = u(z{-}L/2)^2$, where u is a real constant. This potential is presumed to be small compared to the energy E_1 of the lowest state of a simple rectangular potential well of the same width L.
Find an approximate expression, valid in the limit of small u, for the energy difference between the lowest and first excited states of this well in terms of u, L, and fundamental constants.

2. *degenerate perturbation theory*

Consider a cubic quantum box for confining an electron. The cube has length L on all three sides, with edges along the x, y, and z directions, and the walls of the box are presumed to correspond to infinitely high potential barriers. We assume that $(x, y, z) = (0,0,0)$ is the point in the *center* of the box.

i. Write down the normalized wavefunctions for the ground state and the first, threefold-degenerate, excited state for an electron in this box.

ii. Now presume that there is a perturbation $\hat{H}_1 = eFx$ applied (e.g., from an electric field F in the x direction). How does the energy of the ground-state and of the three-fold degenerate excited state change as a result of this perturbation, according to first-order degenerate perturbation theory?

iii. Now presume that a perturbation $\hat{H}_1 = \alpha x^2$ is applied instead. (Such a perturbation could result, e.g., from a uniform fixed background charge density in the box.) Using first-order degenerate perturbation theory, what are the new eigenstates and eigenenergies arising from the three originally degenerate states?

useful integrals:

$$\int_{-\pi/2}^{\pi/2} \theta^2 \cos^2\theta \, d\theta = \pi^3/24 - \pi/4$$

$$\int_{-\pi/2}^{\pi/2} \theta^2 \sin^2 2\theta \, d\theta = \pi^3/24 - \pi/16$$