Exercise Sheet 2 due 31 October

1. *time-evolution in a box*

Consider an electron in an infinite potential well $V(x) = 0$ for $x \in [0, L]$ with initial wave function $\Psi(x, t{=}0) = A(\sin(\pi x/L))^3$

i. Check that $\Psi(x, 0)$ fulfills the boundary conditions $\Psi(0, 0) = 0 = \Psi(L, 0)$ and determine A so that the wave function is normalized.

ii. Solve the initial value problem for the time-dependent Schrödinger equation and show that $\Psi(x, t)$ is normalized for all times t. (Hint: $4(\sin(x))^3 = 3\sin(x) - \sin(3x)$)

iii. Calculate the probability density $|\Psi(x, t)|^2$ and plot it as a function of time. Find the smallest time $T > 0$ for which

$$|\Psi(x, t{+}T)|^2 = |\Psi(x, t)|^2.$$

Write T in terms of the ground state energy of the infinite potential well.

2. *plane wave*

Determine the probability density and the probability current density for an (un-normalized) plane wave $\Psi(\vec{r}, t) = e^{i(\vec{k}\cdot\vec{r}-\omega(\vec{k})t)}$ and verify the continuity equation. The plane wave is an eigenstate of the Hamiltonian $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$. Is it a stationary state?
How do the results depend on the normalization?

3. *wave packet*

i. Find the time it takes a Gaussian wave packet of initial with σ_x to double. Calculate that time for an electron with $\sigma_x = 1$ Å as well for a dust particle of mass 1 mg and width 1 µm.

ii. Make a plot of the probability density for a Gaussian wave packet at different times and observe its group velocity and broadening.