The Slave-Boson Technique

R. Frésard

Laboratoire Crismat, UMR CNRS-ENSICAEN (ISMRA) 6508, Caen, France

Jülich, September 2015

Outline

- ☞ Motivation
- ☞ Review of slave boson approaches: From SIAM to extended Hubbard
- ☞ Radial slave bosons
- ☞ Mott transition
- ☞ Landau parameters
- ☞ Spin and charge instabilites
- ☞ Charge dynamics and upper Hubbard band
- ☞ Summary and outlook

Jülich, September 2015

Fundamental questions and applications

Traditional industrial applications: Cu, steels, Si, plastics, …

- ☞ Mott Transition
- ☞ Quantum critical points
- ☞ Spin-charge separation
- ☞ Competing Instabilities
- ☞ High T_c superconductivity
- ☞ Stripes
- ☞ Colossal magnetoresistance
- ☞ Magnetization steps
- ☞ …

- ☞ Thermoelectricity
- ☞ CMR and magnetic recording
- ☞ Transparent conductors
- ☞ Ferroelectricity and multiferroics
- ☞ Mott FET
- ☞ Superconductors and superconductor based electronics
- ☞ Batteries and solid oxide fuel cells
- ☞ Solar cells
- ☞ …

Needed: tools to investigate these topics (exp. and th.).

Motivation: Hubbard Model, ferromagnetism and charge instabilities

- ☞ The Hubbard model has been initially introduced, *inter alia*, to describe metallic magnetism. Hubbard, Kanamori, Gutzwiller

$$H = \sum_{i,j,\sigma} t_{ij} a^\dagger_{i\sigma} a_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right)$$

- ☞ From Stoner criterion a ferromagnetic instability develops for sufficiently large U.
- ☞ Large strong coupling corrections usually suppress this instability.
- ☞ No ferromagnetism in the Hubbard model on the cubic lattice at half-filling.
- ☞ Connection with the Single Impurity Anderson Model in the DMFT context.

- Role of longer-ranged Coulomb interaction ?
- Magnetic and charge instabilities. Charge dynamics.

Needed: an approach that captures interaction effects beyond the physics of Slater determinants.

Slave boson approaches to strongly interacting fermions

Strategy: Introduce constrained auxiliary particles in order to:

- work with actions that are bi-linear in fermionic fields.
- map degrees of freedom onto bosons (Radial slave bosons).

☞ Barnes: $U = \infty$ Single impurity Anderson model $a_\sigma = e^\dagger f_\sigma$

S. E. Barnes, J. Phys. F **6** 1375 (1976)
P. Coleman, PRB **29** 3035 (1984)

☞ Kotliar and Ruckenstein: Hubbard model

G. Kotliar, A. Ruckenstein, PRL **57** 1362 (1986)

☞ Wölfle *et al.*: Rotationally invariant formulations (t–J model)

RF, P. Wölfle, Int. J. Mod. Phys. B **6** 685 (1992)

☞ Kotliar *et al.*: Multiband models

RF, G. Kotliar, PRB **56**, 12 909 (1997)

☞ ...

Read and Newns: The phase of the slave boson can be gauged away.
Bosonic field x without its phase degree of freedom. Artillery?

N. Read, D. M. Newns, J. Phys. C **16**, L1055 (1983)

- What is the proper functional integral representation for such a field?
- Bose condensation? $\langle e \rangle$ vs $\langle x \rangle$

☞ Singlets?

☞ Entanglement?

Slave boson approaches to strongly interacting fermions

Single impurity Anderson model

Diagrammatic techniques	Green's function of the SIAM	S. Kirchner, J. Kroha, P. Wölfle, PRB **70** 165102 (2004)
		K. Baumgartner, H. Keiter, phys stat sol (b) **242** 377 (2005)

Hubbard model

Saddle-point	Magnetic phases	L. Lilly, A. Muramatsu, and W. Hanke, PRL **65**, 1379 (1990)
Saddle-point	Stripes	G. Seibold, E. Sigmund, V. Hizhnyakov, PRB **57** 6937 (1998)
		M. Raczkowski, RF, A. M. Oleś, PRB **73** 174525 (2006)
Saddle-point	Interfaces	N. Pavlenko, Th. Kopp, PRL **97** 187001 (2006)
Fluctuations	Correlation functions	M. Lavagna, PRB **41** 142 (1990)
Fluctuations	Correlation functions	Y. Bang, C. Castellani, M. Grilli, G. Kotliar, R. Raimondi, and Z. Wang, Int. J. Mod. Phys. B **6**, 531 (1992)
Fluctuations	Structure factors	W. Zimmermann, RF, P. Wölfle, PRB **56** 10 097 (1997)
	Structure factors	E. Koch, PRB **64** 165113 (2001)
	Landau parameters	G. Lhoutellier, RF, A. M. Oleś, PRB **91** 224410 (2015)
	Charge dynamics	V. H. Dao, RF (2016)

Slave boson approaches to strongly interacting fermions

Consider the single impurity Anderson model:

$$H = \sum_{\vec{k}\sigma} \varepsilon_{\vec{k}} c^{\dagger}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \varepsilon_f \sum_{\sigma} a^{\dagger}_{\sigma} a_{\sigma} + V \sum_{\vec{k}\sigma} \left(c^{\dagger}_{\vec{k}\sigma} a_{\sigma} + a^{\dagger}_{\sigma} c_{\vec{k}\sigma} \right) + U a^{\dagger}_{\uparrow} a_{\uparrow} a^{\dagger}_{\downarrow} a_{\downarrow}$$

in the $U \to \infty$ limit. Barnes introduced the auxiliary fermionic (f_σ) and bosonic (e) operators in terms of which the physical electron operators a_σ read,

$$a_\sigma = e^{\dagger} f_\sigma$$

The a_σ-operators obey the ordinary Fermion anticommutation relations. Not automatically preserved when using this representation, even when the auxiliary operators obey canonical commutation relations. In addition there is a constraint that must be satisfied:

$$Q \equiv e^{\dagger} e + \sum_{\sigma} f^{\dagger}_{\sigma} f_{\sigma} = 1$$

A faithful representation of the physical electron operator is obtained in the sense that both have the same matrix elements in the physical Hilbert subspace with $Q = 1$.

Operators : $\hat{n} = 1 - e^{\dagger} e$ $\qquad \hat{\mathbf{S}} = \frac{1}{2} \sum_{\sigma,\sigma'} f^{\dagger}_{\sigma} \tau_{\sigma,\sigma'} f_{\sigma'}$ $\qquad$ Asymmetry

Slave boson approaches to strongly interacting fermions: Implementation

For the single impurity Anderson model:

$$H = \sum_{\vec{k}\sigma} \varepsilon_{\vec{k}} c^{\dagger}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \varepsilon_f \sum_{\sigma} a^{\dagger}_{\sigma} a_{\sigma} + V \sum_{\vec{k}\sigma} \left(c^{\dagger}_{\vec{k}\sigma} a_{\sigma} + a^{\dagger}_{\sigma} c_{\vec{k}\sigma} \right) + U a^{\dagger}_{\uparrow} a_{\uparrow} a^{\dagger}_{\downarrow} a_{\downarrow}$$

and $U \to \infty$, the partition function, projected onto the $Q = 1$ subspace, reads,

$$\begin{aligned} Z \quad = \quad & \int_{-\pi/\beta}^{\pi/\beta} \frac{\beta d\lambda}{2\pi} e^{i\beta\lambda} \int D[e, e^{\dagger}] \\ & \int \prod_{\sigma} D[f_{\sigma}, f^{\dagger}_{\sigma}] \int \prod_{\vec{k}\sigma} D[c_{\vec{k}\sigma}, c^{\dagger}_{\vec{k}\sigma}] \mathrm{e}^{-\int_0^{\beta} \mathrm{d}\tau (\mathcal{L}_{\mathrm{f}}(\tau) + \mathcal{L}_{\mathrm{b}}(\tau))} \end{aligned}$$

with the fermionic and bosonic Lagrangians

$$\begin{aligned} \mathcal{L}_f(\tau) \quad &= \quad \sum_{\vec{k}\sigma} c^{\dagger}_{\vec{k}\sigma}(\tau)(\partial_{\tau} + \varepsilon_{\vec{k}} - \mu) c_{\vec{k}\sigma}(\tau) + \sum_{\sigma} f^{\dagger}_{\sigma}(\tau)(\partial_{\tau} + \varepsilon_f - \mu + i\lambda) f_{\sigma}(\tau) \\ & + \quad V \sum_{\vec{k}\sigma} \left(c^{\dagger}_{\vec{k}\sigma}(\tau) f_{\sigma}(\tau) e^{\dagger}(\tau) + h.\, c. \right) \\ \mathcal{L}_b(\tau) \quad &= \quad e^{\dagger}(\tau)(\partial_{\tau} + i\lambda) e(\tau) \end{aligned}$$

Here the λ integration enforces the constraint, and the Lagrangian is bi-linear in the fermionic fields. This has been achieved without decoupling the interaction term.

Slave boson approaches to strongly interacting fermions: Implementation

For the $U \to \infty$ single impurity Anderson model the partition function may be written as:

$$Z = \int_{-\pi/\beta}^{\pi/\beta} \frac{\beta d\lambda}{2\pi} e^{i\beta\lambda} \int D[e, e^{\dagger}] \mathrm{e}^{-\int_0^\beta \mathrm{d}\tau \mathcal{L}_{\mathrm{b}}(\tau)} \mathrm{Z}_{\mathrm{f}}^2$$

$$Z_f = \det \begin{pmatrix} \mathbb{1}_2 & & & -[\mathcal{L}_1] \\ [\mathcal{L}_2] & \mathbb{1}_2 & & \\ & \ddots & \ddots & \\ & & [\mathcal{L}_{\mathrm{N}}] & \mathbb{1}_2 \end{pmatrix}$$

where $\mathbb{1}_2$ is the 2×2 identity matrix and $[\mathcal{L}_n]$ are (2×2) blocks given, in the simplest limit, by:

$$[\mathcal{L}_n] = \begin{pmatrix} -L_{\mathrm{c}} & \delta V e_n^{\dagger} \\ \delta V e_{n-1} & -L_f \end{pmatrix}$$

where $L_{\mathrm{c}} = e^{-\delta(\epsilon_{\mathrm{c}}-\mu)}$, $L_f = e^{-\delta(\epsilon_{\mathrm{f}}-\mu+i\lambda)}$. It makes sense to diagonalize $[\mathcal{L}_n]$ in a saddle-point approximation.

Path integral representation of slave bosons in radial gauge

One can make use of $a_\sigma^\dagger \rightarrow e f_\sigma^\dagger$ to retain the sole amplitude of the slave boson field x. One may then write the partition sum for a lattice problem on a discretized time mesh as:

$$Z = \lim_{\substack{N\to\infty \\ W\to\infty}} \left(\prod_n \int_{-\infty}^{\infty} \frac{\delta d\lambda_n}{2\pi} \int_{-\infty}^{\infty} dx_n \int \prod_\sigma D[f_{n,\sigma}, f_{n,\sigma}^\dagger] D[c_{n,\sigma}, c_{n,\sigma}^\dagger] \right) e^{-S} \quad \text{with}$$

$$S = S_f + S_b + S_V \quad \text{where} \quad \delta = \frac{\beta}{N} \quad \text{and}$$

$$S_f = \sum_{n,\sigma} \left[f_{n,\sigma}^\dagger \left(f_{n,\sigma} - f_{n-1,\sigma} e^{-\delta(i\lambda_n + \varepsilon_f - \mu)} \right) + c_{n,\sigma}^\dagger \left(c_{n,\sigma} - c_{n-1,\sigma} e^{-\delta(-\mu)} \right) \right]$$

$$S_b = \delta \sum_n \left(i\lambda_n (x_n - 1) + W x_n (x_n - 1) \right)$$

$$S_V = \delta V \sum_{n,\sigma} x_n \left(c_{n,\sigma}^\dagger f_{n-1,\sigma} + f_{n,\sigma}^\dagger c_{n-1,\sigma} \right)$$

Used: $a_{n,\sigma}^\dagger = x_n f_{n,\sigma}^\dagger$; $a_{n,\sigma} = x_{n+1} f_{n,\sigma}$.

Here the measure is trivial, and the interaction terms included in S_b are bilinear. The "W-term" allows the amplitudes for running from $-\infty$ to $+\infty$.

RF and T. Kopp, Nucl. Phys. B **94**, 769 (2001)

Exact evaluation of path integral representations involving slave bosons in radial gauge

After some algebra: $\mathcal{Z} = \lim_{\substack{N\to\infty \\ W\to\infty}} \mathcal{P}_1 \ldots \mathcal{P}_N \left(\mathrm{Tr} \prod_{n=1}^{N} [\mathcal{K}_n] \otimes [\mathcal{K}_n] \right)$

With $\mathcal{P}_n = \int_{-\infty}^{\infty} \frac{\delta d\lambda_n}{2\pi} \int_{-\infty}^{\infty} dx_n e^{-\delta(i\lambda_n(x_n-1)+Wx_n(x_n-1))}$

And $[\mathcal{K}_n] = \begin{pmatrix} 1 & & & \\ & L_c & \delta V x_n & \\ & \delta V x_n & L_n & \\ & & & L_c L_n \end{pmatrix}$, $L_c = e^{-\delta(\epsilon_c-\mu)}$, $L_n = e^{-\delta(\epsilon_f-\mu+i\lambda_n)}$

The time steps are decoupled!

$\rightarrow$ block diagonal Hamiltonian matrix including entangled states.

$$\mathcal{Z}\langle x_m \rangle = \mathcal{P}_1 \ldots \mathcal{P}_N \left(x_m \mathrm{Tr} \prod_{n=1}^{N} [\mathcal{K}_n] \otimes [\mathcal{K}_n] \right)$$

One finds $\langle x_m \rangle$ to be generically finite and not related to a Bose condensate.
A saddle-point approximation yields an approximate value to $\langle x_m \rangle$.

RF, H. Ouerdane, and T. Kopp, Nucl. Phys. B **785**, 286 (2007)

Extension to the Kotliar and Ruckenstein representation

Principle: Introduce the auxiliary particles f_σ, e, p_σ, and d to represent the physical states:

$$
\begin{aligned}
|0\rangle &= e^\dagger|\mathrm{vac}\rangle \\
|\sigma\rangle &= p_\sigma^\dagger f_\sigma^\dagger|\mathrm{vac}\rangle \\
|2\rangle &= d^\dagger f_\uparrow^\dagger f_\downarrow^\dagger|\mathrm{vac}\rangle
\end{aligned}
$$

There are now three constraints:

$$
\begin{aligned}
1 &= e^\dagger e + \sum_\sigma p_\sigma^\dagger p_\sigma + d^\dagger d \\
f_\sigma^\dagger f_\sigma &= p_\sigma^\dagger p_\sigma + d^\dagger d\,,\ \sigma = \uparrow,\downarrow
\end{aligned}
$$

which are implemented in path integral.

The phase of 3 slave boson fields may be gauged away.

Operators: $a_\sigma^\dagger = \tilde{z}_\sigma^\dagger f_\sigma^\dagger$ with $\tilde{z}_\sigma^\dagger = p_\sigma^\dagger e + d^\dagger p_{-\sigma}$. Problematic. Yet, with:

$$\tilde{z}_\sigma^\dagger = p_\sigma^\dagger L_\sigma R_\sigma e + d^\dagger L_\sigma R_\sigma p_{-\sigma}\,;\ L_\sigma \equiv \left(1 - d^\dagger d - p_\sigma^\dagger p_\sigma\right)^{-\frac{1}{2}};\ R_\sigma \equiv \left(1 - e^\dagger e - p_{-\sigma}^\dagger p_{-\sigma}\right)^{-\frac{1}{2}}$$

the Gutzwiller approximation is recovered as a saddle-point.

Operators : $\hat{n} = \sum_\sigma p_\sigma^\dagger p_\sigma + 2d^\dagger d$ $\qquad \hat{\mathbf{S}} = \frac{1}{2}\sum_{\sigma,\sigma'} a_\sigma^\dagger \tau_{\sigma,\sigma'} a_{\sigma'}$ $\qquad \hat{D} = d^\dagger d$

Asymmetry

G. Kotliar, A. Ruckenstein, PRL **57** 1362 (1986)

Extension to the Spin Rotation Invariant Kotliar and Ruckenstein representation

Introduce the auxiliary canonical particles f_σ, e, p_0, $\vec{p}$, and d to represent the physical states:

$$
\begin{aligned}
|0\rangle &= e^\dagger|\text{vac}\rangle \\
|\sigma\rangle &= \sum_{\sigma'} p^\dagger_{\sigma\sigma'} f^\dagger_{\sigma'}|\text{vac}\rangle \quad \text{with} \quad p^\dagger_{\sigma\sigma'} = \frac{1}{2}\sum_{\mu=0,x,y,z} p^\dagger_\mu \tau^\mu_{\sigma\sigma'} \\
|2\rangle &= d^\dagger f^\dagger_\uparrow f^\dagger_\downarrow|\text{vac}\rangle
\end{aligned}
$$

There are now five constraints:

$$
\begin{aligned}
1 &= e^\dagger e + \sum_\mu p^\dagger_\mu p_\mu + d^\dagger d \\
\sum_\sigma f^\dagger_\sigma f_\sigma &= \sum_\mu p^\dagger_\mu p_\mu + 2d^\dagger d \\
\sum_{\sigma,\sigma'} f^\dagger_\sigma \vec{\tau}_{\sigma,\sigma'} f_{\sigma'} &= p^\dagger_0 \vec{p} + \vec{p}^{\,\dagger} p_0 - i\vec{p}^{\,\dagger} \times \vec{p}
\end{aligned}
$$

which are implemented in path integral.

The phases of 5 bosons may be gauged away.

Fermionic operators : $a_\sigma = \sum_{\sigma'} f_{\sigma'} z_{\sigma',\sigma}$ with $\underline{z} = e^\dagger \underline{L}\; M\; \underline{R}\; \underline{p} + \underline{\tilde{p}}^\dagger \underline{L}\; M\; \underline{R}\; d$, and

$$M = \left[1 + e^\dagger e + \sum_\mu p^\dagger_\mu p_\mu + d^\dagger d\right]^{\frac{1}{2}} \quad \underline{L} = \left[\left(1 - d^\dagger d\right)\underline{1} - 2\underline{p}^\dagger \underline{p}\right]^{-\frac{1}{2}} \quad \underline{R} = \left[\left(1 - e^\dagger e\right)\underline{1} - 2\underline{\tilde{p}}^\dagger \underline{\tilde{p}}\right]^{-\frac{1}{2}}$$

Extension to the Spin Rotation Invariant Kotliar and Ruckenstein representation

Operators : $\hat{n} = \sum_\mu p^\dagger_\mu p_\mu + 2d^\dagger d \qquad \vec{S} = \sum_{\sigma\sigma'\sigma_1} \vec{\tau}_{\sigma\sigma'} p^\dagger_{\sigma\sigma_1} p_{\sigma_1\sigma'} \qquad \hat{D} = d^\dagger d$

All these degrees of freedom have been mapped onto bosons.

Kinetic energy : $\hat{T} = \sum_{i,j} t_{i,j} \sum_{\sigma,\sigma',\sigma_1} z^\dagger_{i,\sigma,\sigma_1} f^\dagger_{i,\sigma_1} f_{j,\sigma'} z_{j,\sigma',\sigma}$

Fermion–boson interaction term.

At saddle-point in the paramagnetic phase the action for the Hubbard model reads:

$$S = \beta L \left(-\frac{1}{\beta} \sum_{\vec{k},\sigma} \ln\left(1 + e^{-\beta E_{\vec{k}\sigma}}\right) + Ud^2 + \alpha(e^2 + d^2 + p_0^2 - 1) - \beta_0(p_0^2 + 2d^2) \right)$$

The quasiparticle dispersion reads:

$$E_{\vec{k}\sigma} = z_0^2 t_{\vec{k}} + \beta_0 - \mu$$

The Gutzwiller approximation is recovered as a saddle-point.

Large N theory.

G. Kotliar, A. Ruckenstein, PRL **57** 1362 (1986)

RF, P. Wölfle, Int. J. Mod. Phys. B **6** 685 (1992)

SRI Kotliar and Ruckenstein representation: saddle-point approximation

At the paramagnetic saddle-point ($\vec{p} = \vec{\beta} = 0$). Saddle-point equations arise as:

$$\begin{aligned}
p_0^2 + e^2 + d^2 - 1 &= 0, \\
p_0^2 + 2d^2 &= n, \\
\frac{1}{2e}\frac{\partial z_0^2}{\partial e}\bar{\varepsilon} &= -\alpha, \\
\frac{1}{2p_0}\frac{\partial z_0^2}{\partial p_0}\bar{\varepsilon} &= \beta_0 - \alpha, \\
\frac{1}{2d}\frac{\partial z_0^2}{\partial d}\bar{\varepsilon} &= 2(\beta_0 - \alpha) + \alpha - U,
\end{aligned}$$

with $y \equiv (e+d)^2$, $z_0 = \left(\frac{y p_0^2}{2n_\sigma(1-n_\sigma)}\right)^{\frac{1}{2}}$, and $\bar{\varepsilon} = \int d\omega \rho(\omega)\omega f_F\left(z_0^2\omega + \beta_0 - \mu\right)$, the saddle-point equation may be written as:

$$y^3 + (u-1)y^2 = u\delta^2\,, \text{ where } u = U/U_0, \text{ and } U_0 = -\frac{8}{1-\delta^2}\,\bar{\varepsilon}.$$

At half filling one finds $y = 1 - u$, and a metal-to-insulator transition occurs at

$$U_c = \lim_{\delta \to 0} U_0 = -8\bar{\varepsilon}.$$

G. Kotliar, A. Ruckenstein, PRL **57** 1362 (1986)

SRI representation of an extended Hubbard Model

$$
\begin{aligned}
H &= \sum_{i,j,\sigma} t_{ij} a^{\dagger}_{i\sigma} a_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} \\
&+ \frac{1}{2} \sum_{i,j} V_{ij} (1-n_i)(1-n_j) + \frac{1}{2} \sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j
\end{aligned}
$$

It includes local Coulomb U, intersite Coulomb V_{ij} and exchange J_{ij} interactions.
Imperfect screening.
The Spin Rotation Invariant representation of this Hamiltonian may be written as:

$$
\begin{aligned}
H &= \sum_{i,j,\sigma} t_{i,j} \sum_{\sigma\sigma'\sigma_1} z^{\dagger}_{i\sigma_1\sigma} f^{\dagger}_{i\sigma} f_{j\sigma'} z_{j\sigma'\sigma_1} + U \sum_{i} d^{\dagger}_i d_i \\
&+ \frac{1}{4} \sum_{i,j} V_{ij} \left[\left(1 - \sum_{\sigma} f^{\dagger}_{i\sigma} f_{i\sigma} \right) Y_j + Y_i \left(1 - \sum_{\sigma} f^{\dagger}_{j\sigma} f_{j\sigma} \right) \right] \\
&+ \frac{1}{2} \sum_{i,j} J_{ij} \sum_{\sigma\sigma'\sigma_1} \vec{\tau}_{\sigma\sigma'} p^{\dagger}_{i\sigma\sigma_1} p_{i\sigma_1\sigma'} \cdot \sum_{\rho\rho'\rho_1} \vec{\tau}_{\rho\rho'} p^{\dagger}_{j\rho\rho_1} p_{j\rho_1\rho'} \\
Y_i &\equiv e^{\dagger}_i e_i - d^{\dagger}_i d_i \qquad \text{Symmetric form}
\end{aligned}
$$

Mott transition

All saddlepoint equations equations may be merged into a single one:

$$y^3 + (u-1)y^2 = u\delta^2$$

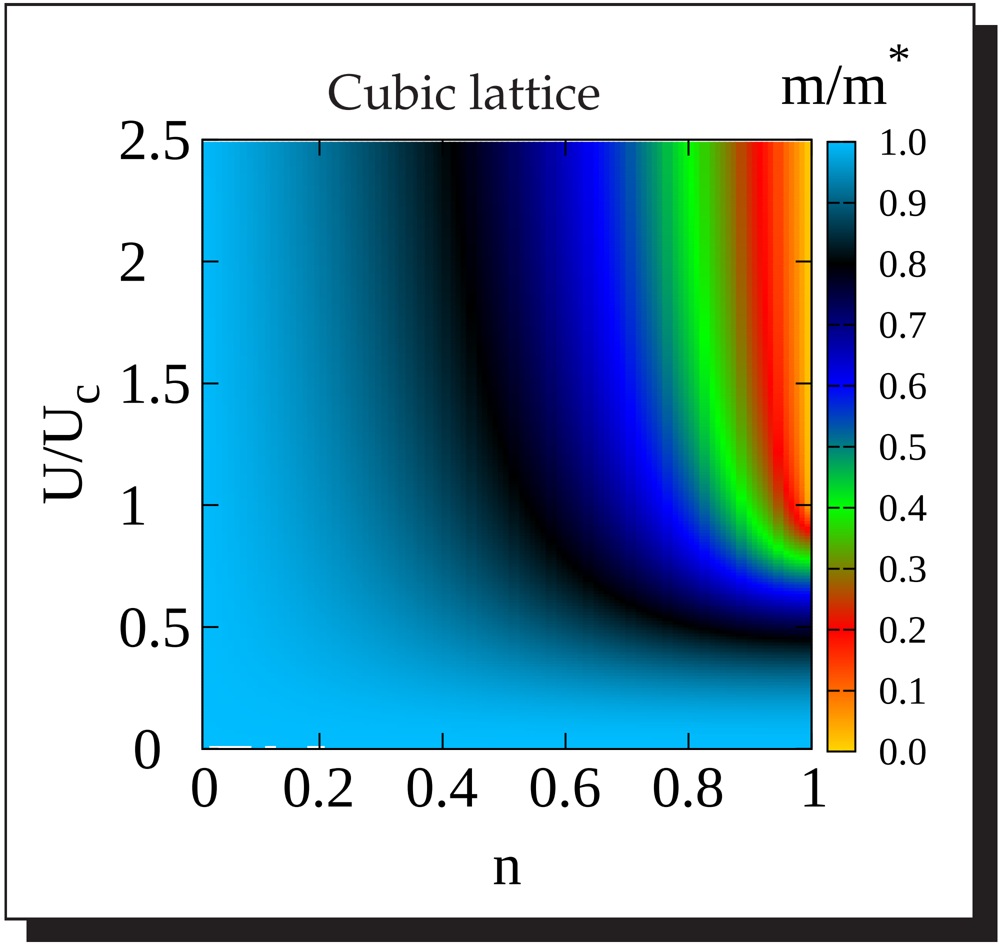
with:

$$y = (e+d)^2; u = U/U_c$$

V and J disappear!

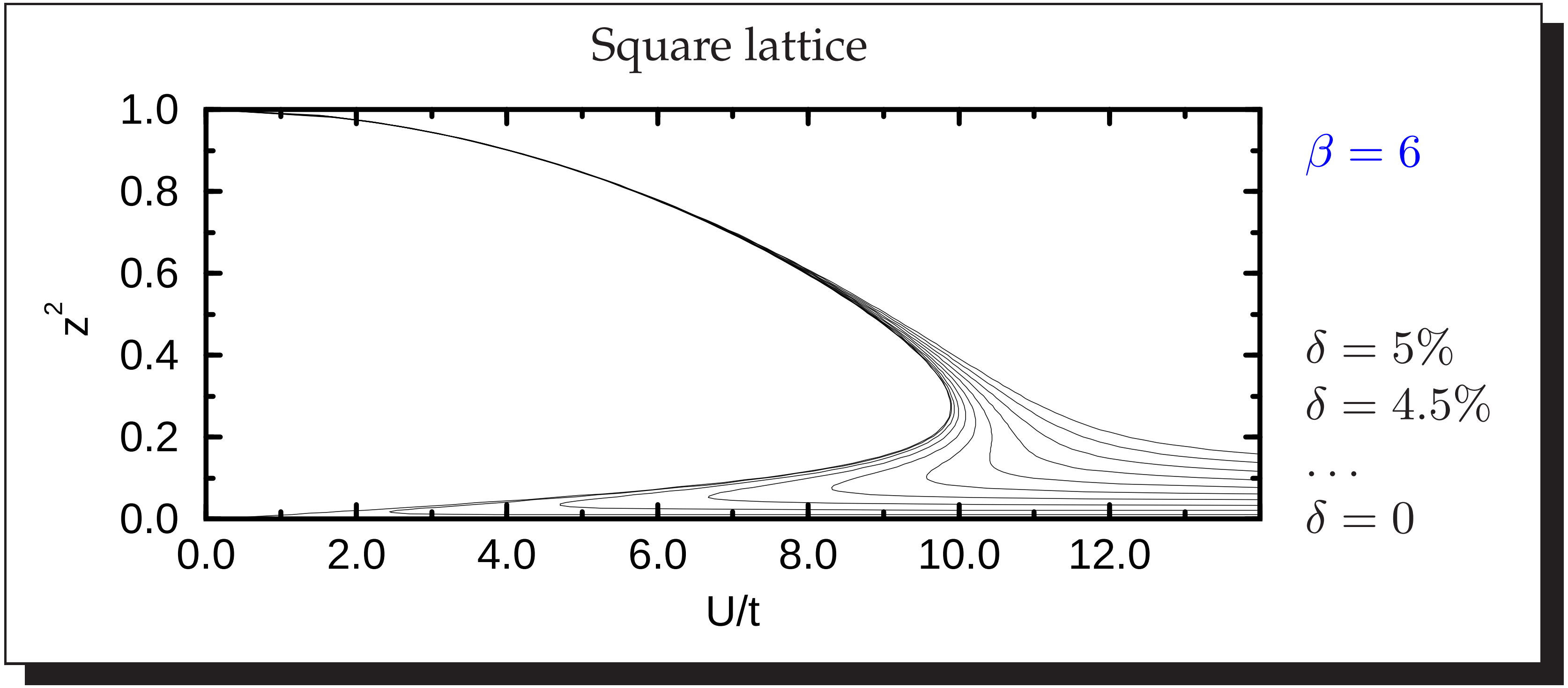
$$z^2 = 1 - \left(\frac{U}{U_c}\right)^2$$

Mott gap as in the plane Hubbard Model



G. Lhoutellier, RF, and A. M. Oleś, PRB **91** 224410 (2015)

Mott transition at finite temperature



First order transition line

Its length increases with the temperature

RF and K. Doll, 1995 *(arXiv:cond-mat/9603125)*

Susceptibilities

Having mapped all degrees of freedom onto bosons allows to write the spin and density fluctuations as:

$$\begin{aligned}
\delta S_z &\equiv \sum_\sigma \sigma \delta n_\sigma = \delta(p_0^\dagger p_z + p_z^\dagger p_0) \\
\delta N &\equiv \sum_\sigma \delta n_\sigma = \delta(d^\dagger d - e^\dagger e)
\end{aligned}$$

The spin and charge autocorrelation functions can be written in terms of the slave boson correlation functions as:

$$\begin{aligned}
\chi_s(k) &= \sum_{\sigma,\sigma'} \sigma\sigma' \langle \delta n_\sigma(-k)\delta n_{\sigma'}(k)\rangle = \langle \delta S_z(-k)\delta S_z(k)\rangle, \\
\chi_c(k) &= \sum_{\sigma\sigma'} \langle \delta n_\sigma(-k)\delta n_{\sigma'}(k)\rangle = \langle \delta N(-k)\delta N(k)\rangle.
\end{aligned}$$

With $k \equiv (\vec{k}, \omega)$.

- Derive the inverse propagator matrix to compute the one-loop result.
- The spin and charge degrees of freedom are decoupled.

Landau parameters: F_0^a and F_0^s

Spin susceptibility:

$$\chi_s(\mathbf{q},\omega) = \frac{\chi_0(\mathbf{q},\omega)}{1 + A_{\vec{\mathbf{q}}}\chi_0(\mathbf{q},\omega) + B\chi_1(\mathbf{q},\omega) + C[\chi_1^2(\mathbf{q},\omega) - \chi_0(\mathbf{q},\omega)\chi_2(\mathbf{q},\omega)]}$$

with

$$\chi_n(\mathbf{q}, i\nu_n) = -\frac{1}{T}\sum_{\mathbf{p}, i\omega_n, \sigma}(t_{\mathbf{q}} + t_{\mathbf{q}+\mathbf{p}})^n G_{0,\sigma}(\mathbf{p}, i\omega_n) G_{0,\sigma}(\mathbf{q}+\mathbf{p}, i\omega_n + i\nu_n)$$

☞ It takes a form similar to RPA, with an effective interaction.

Landau parameters:

$$\chi_s(0) = \frac{\chi_0(0)}{1+F_0^a}$$

$$\chi_c(0) = \frac{\chi_0(0)}{1+F_0^s}$$

Half-filling:

$$F_0^a = -1 + \frac{1}{(1+\frac{U}{U_c})^2}$$

$$F_0^s = -1 + \frac{1}{(1-\frac{U}{U_c})^2}$$

$F_0^a(F_0^s) = -1$ signals an instability.
They show a lesser sensitivity to the density of states.

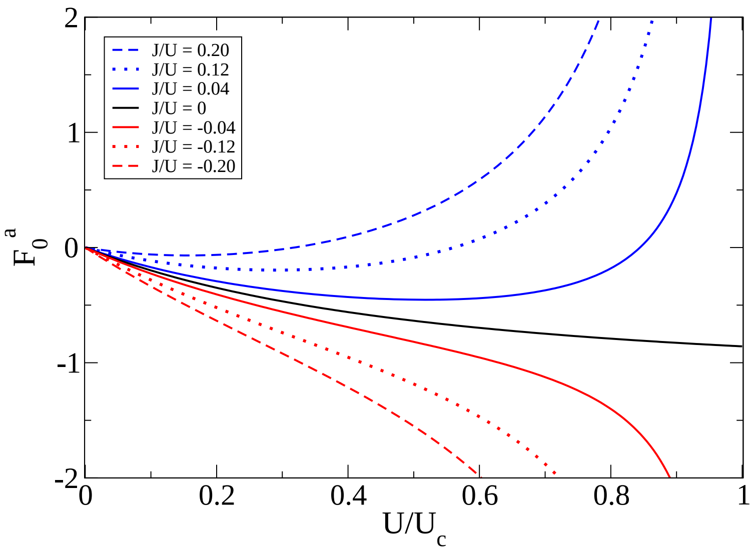
D. Vollhardt, Rev. Mod. Phys. **56**, 99 (1984)

Landau parameter of the extended Hubbard Model: F_0^a at half-filling on the cubic lattice

$$F_0^a = -2N_F^{(0)}\bar{\varepsilon}\left\{\frac{-u(2+u)}{(1+u)^2} + \frac{1}{8}\frac{2+u}{1-u^2}\frac{J_0}{-\bar{\varepsilon}}\right\} \quad \text{with } u \equiv \frac{U}{U_c}$$

Appearance of a singular contribution

$J_{\mathbf{k}=\mathbf{0}}$ only dictates the results

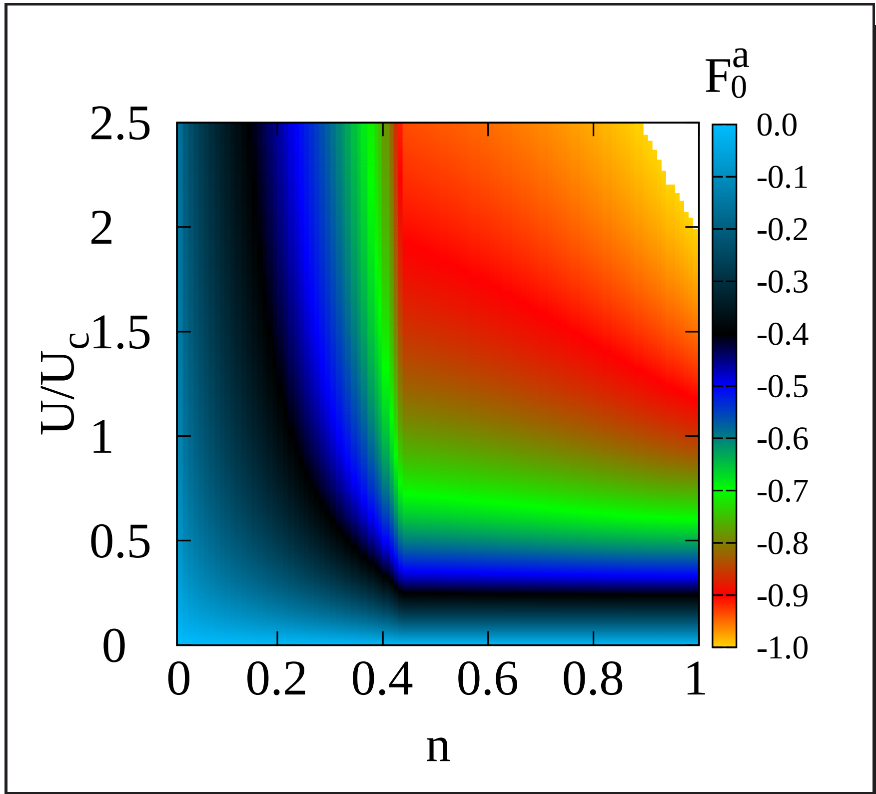


Negative J triggers an instability
Smooth behavior
Positive J: softer and stiffer responses

G. Lhoutellier, RF, and A. M. Oleś, PRB **91** 224410 (2015)

Landau parameter F_0^a on the cubic lattice: Doping dependence

$J/U = 0$

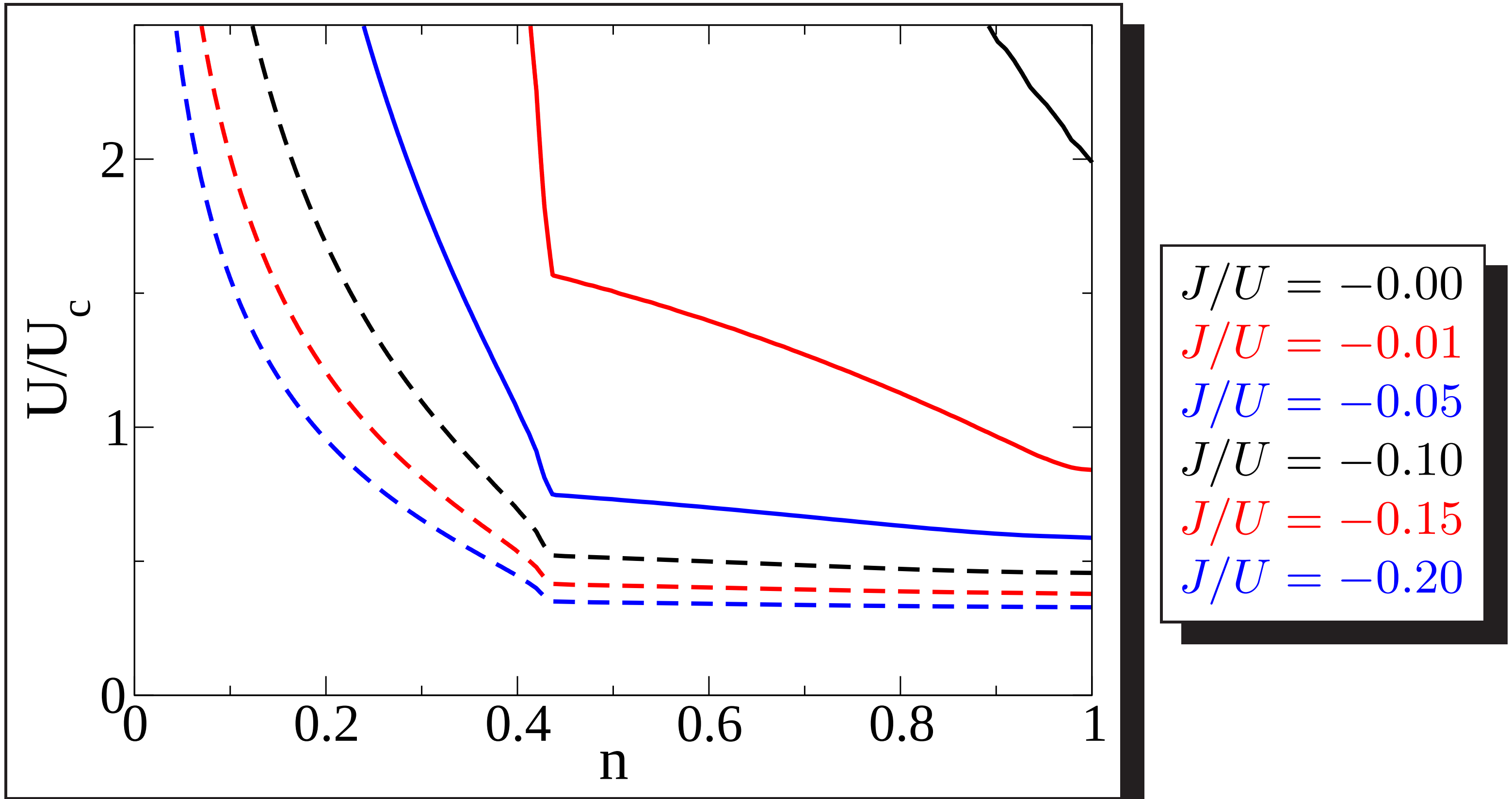


Soft response

Instability for large U and small doping

Landau parameter F_0^a of the extended Hubbard Model: Instabilities of the cubic lattice

Instability lines

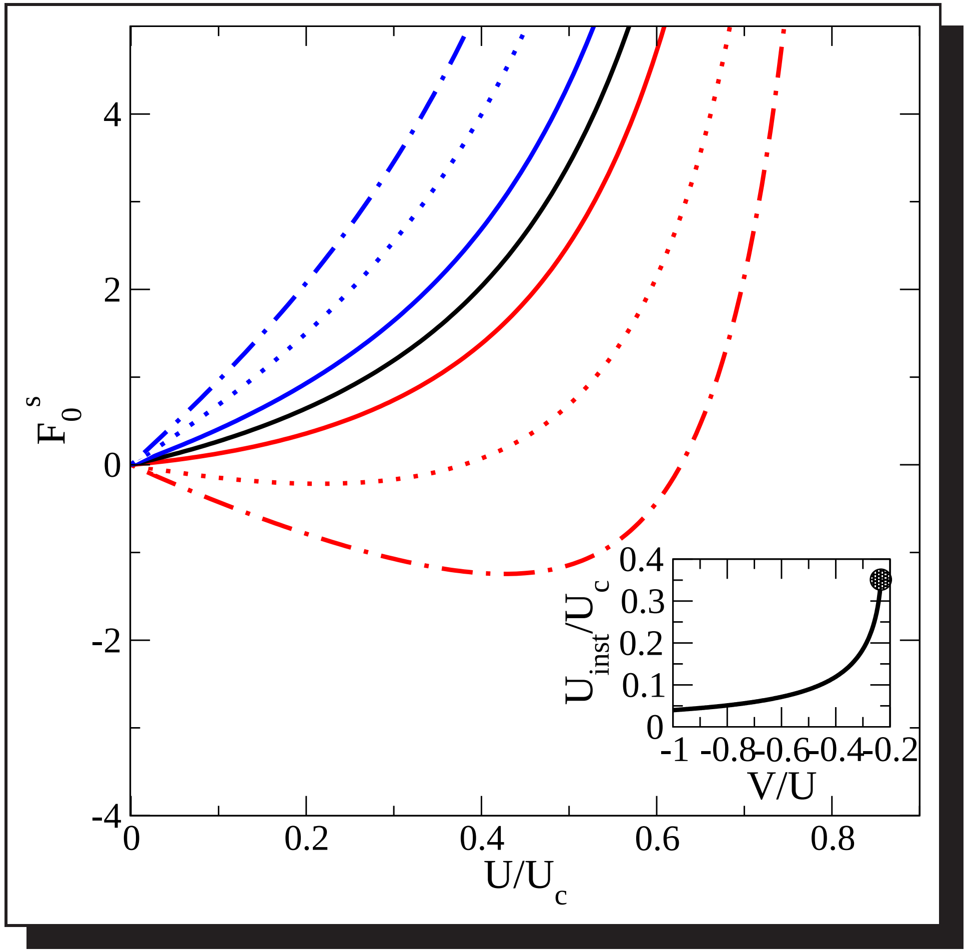


Increasing $-J$ softens the response

Instabilities for $U < U_c$

Instabilities at low densities

F_0^s of the extended Hubbard Model: Instabilities at half-filling on the cubic lattice



$V/U = 0.25$
$V/U = 0.15$
$V/U = 0.05$
$V/U = 0.00$
$V/U = -0.05$
$V/U = -0.15$
$V/U = -0.25$

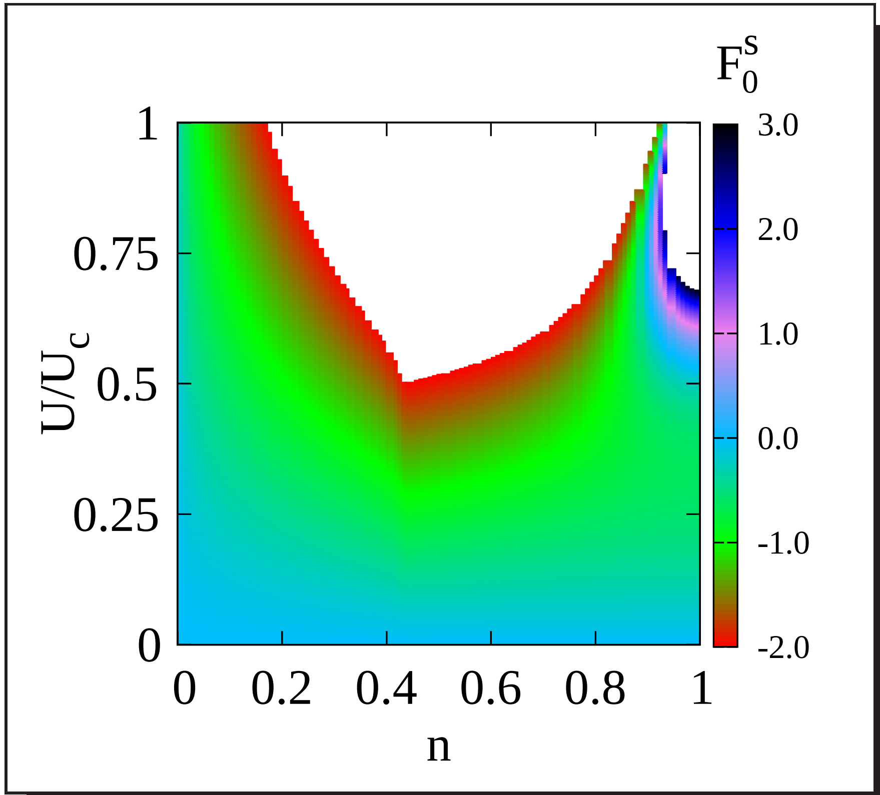
The stiffness increases with V

No instability unless $-V/U > 0.234$

Re-entrant behavior

F_0^s of the extended Hubbard Model: Doping dependence on the cubic lattice

$V/U = -0.2$



Stiff response close to half-filling

Soft response and
instabilities
around quarter-filling

The instability line extends to
$n = 1^-$ and stops at $U \simeq 1.246 U_c$

G. Lhoutellier, RF, and A. M. Oleś, PRB **91** 224410 (2015)

Charge dynamics

The gaussian fluctuation separate into a charge channel and a spin channel. Here:

$$S_c = \sum_q \sum_{\mu,\nu} \delta\psi_\mu(-q) S_{\mu,\nu}(q) \delta\psi_\nu(q)$$

with $\delta\psi_1(q) = \delta e(q)$, $\delta\psi_2(q) = \delta d'(q)$, $\delta\psi_3(q) = \delta d''(q)$, $\delta\psi_4(q) = \delta p_0(q)$, $\delta\psi_5(q) = \delta\beta_0(q)$, $\delta\psi_6(q) = \delta\alpha(q)$. The "main" contribution to $S_{\mu,\nu}(q)$ reads:

$$S^{(1)}(q) = \begin{pmatrix} \alpha & 0 & 0 & 0 & 0 & e \\ 0 & \alpha - 2\beta_0 + U & \nu_n & 0 & 0 & d \\ 0 & -\nu_n & \alpha - 2\beta_0 + U & 0 & -2d & 0 \\ 0 & 0 & 0 & \alpha - \beta_0 & -p_0 & p_0 \\ 0 & -2d & 0 & -p_0 & -\frac{1}{2}\chi_0(q) & 0 \\ e & d & 0 & p_0 & 0 & 0 \end{pmatrix},$$

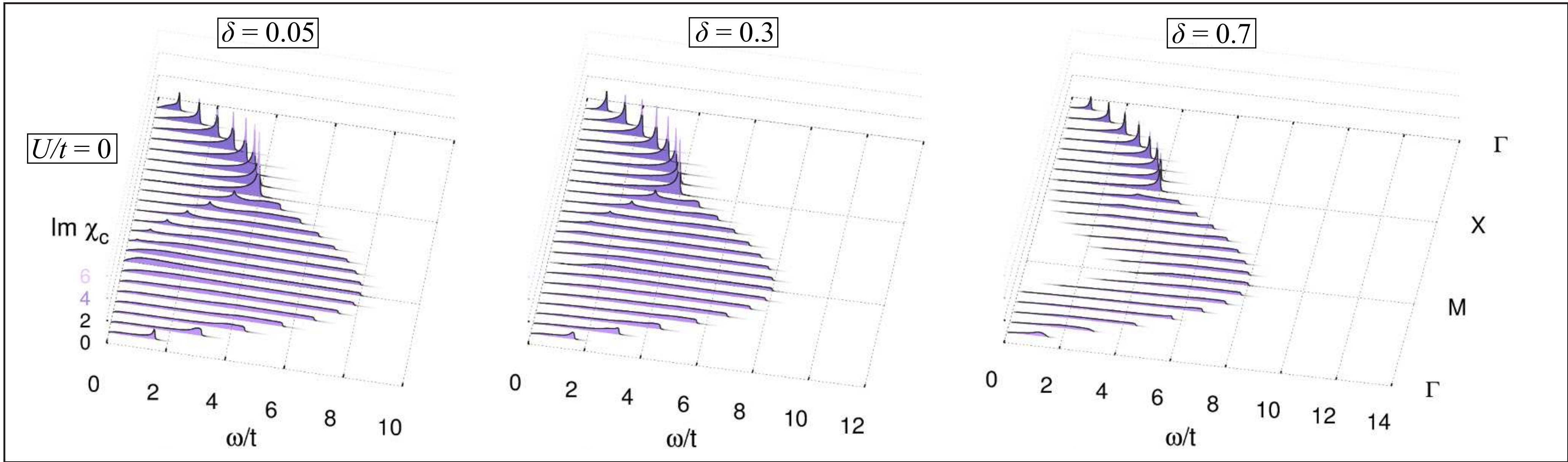
which may be used to obtain the charge susceptibility as:

$$\chi_c(q) = 2e^2 S^{-1}_{11}(q) - 4ed S^{-1}_{12}(q) + 2d^2 S^{-1}_{22}(q).$$

A pole arises in the charge dynamics at $\omega \simeq \alpha - 2\beta_0 + U \simeq U$. Upper Hubbard band?

Charge dynamics on the square lattice

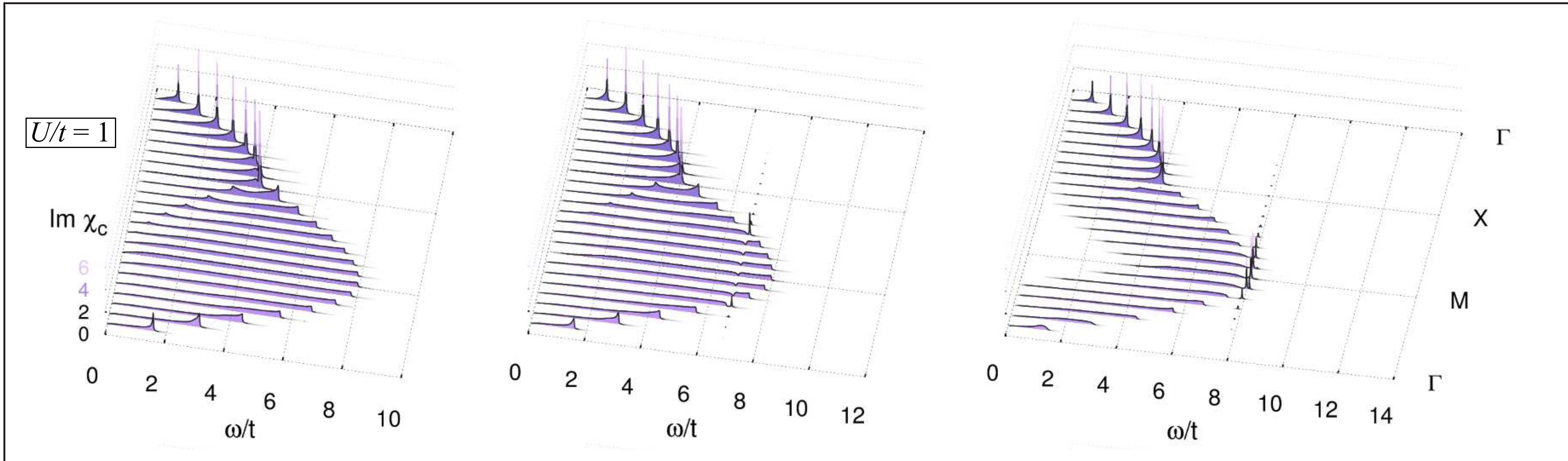
Performing the full one-loop calculation of the charge autocorrelation function yields $\chi_c(\mathbf{q},\omega) = \chi_0(\mathbf{q},\omega)$ in the non-interacting limit. This is a fully interacting problem in slave boson representations.



$U/t = 0$: Particle-hole continuum. Logarithmic singularities along $\Gamma - X$ at $\beta = 8$.

V. H. Dao, RF (2016)

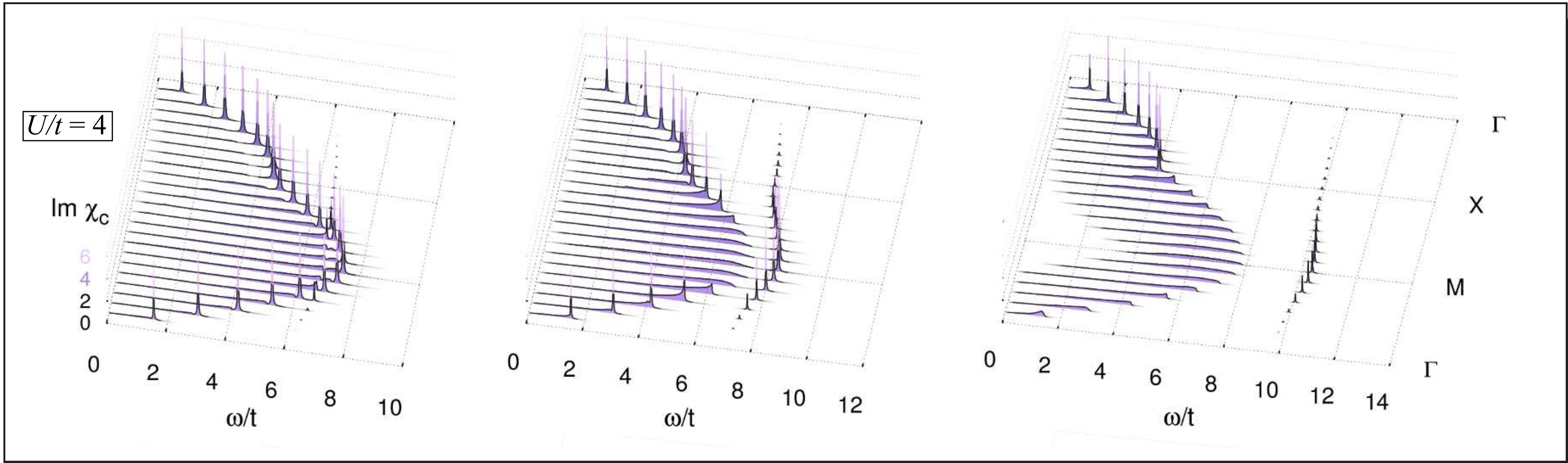
Charge dynamics on the square lattice



$U/t = 1$: The edge peak is complemented by:

- An additional feature in the continuum: Damped collective mode
- A dispersionless mode

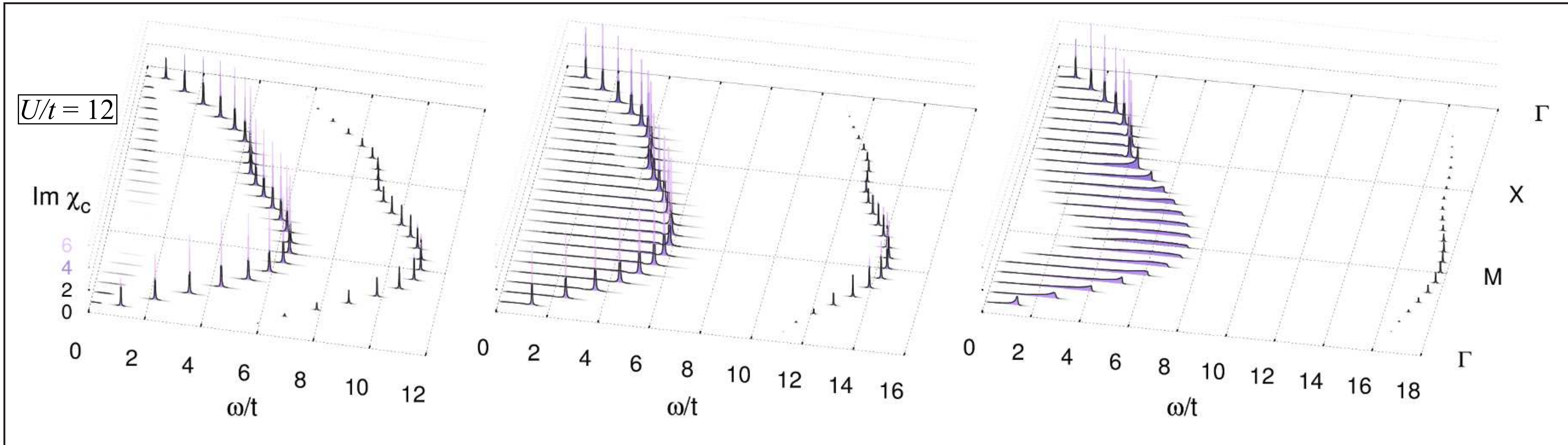
Charge dynamics on the square lattice



$U/t = 4$:

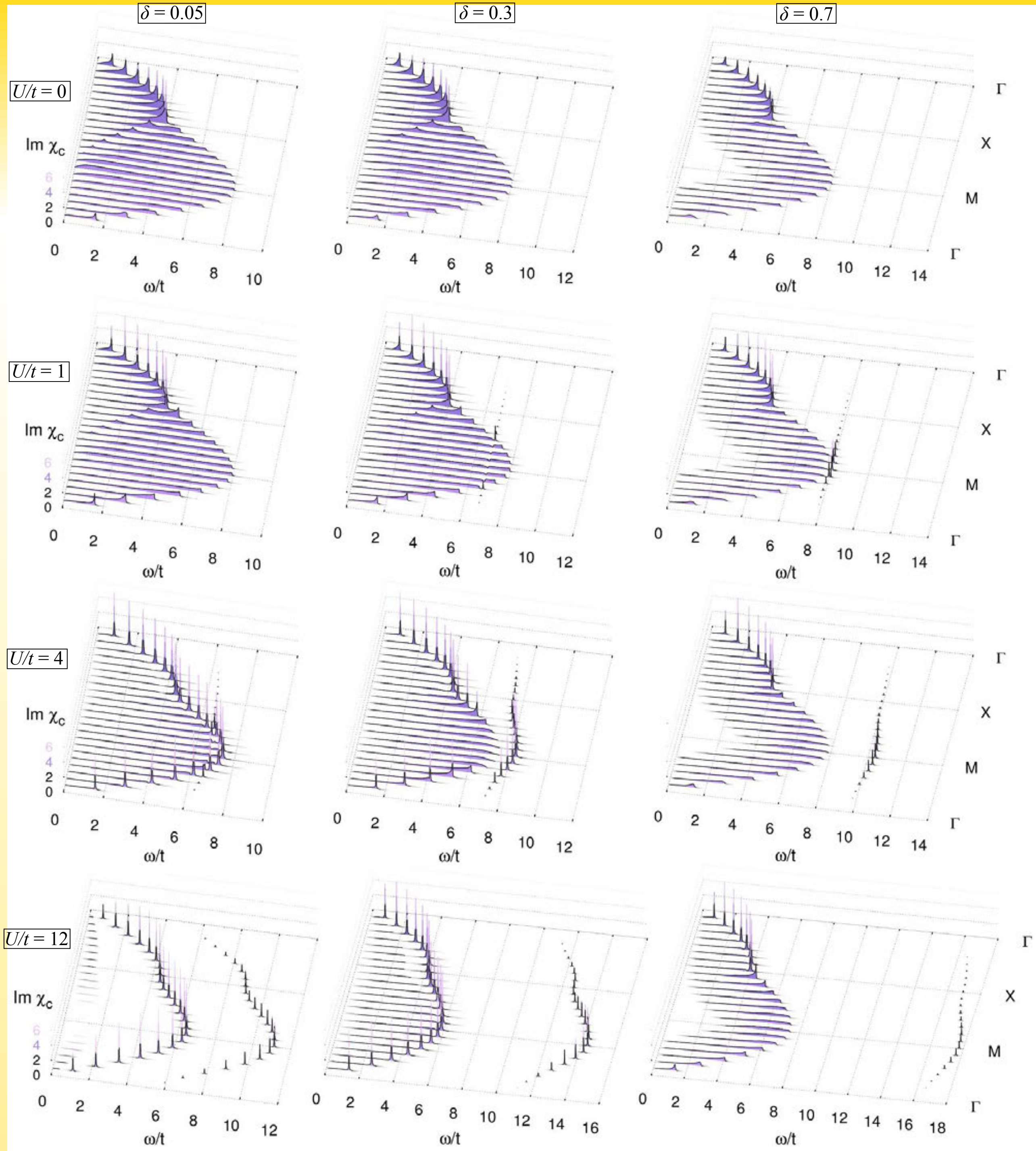
- The particle-hole continuum shrinks due to mass renormalization
- The collective mode separates from the continuum
- The upper mode acquires weight and dispersion. δ dependence

Charge dynamics on the square lattice



$U/t = 12$:

- The particle-hole continuum further shrinks due to mass renormalization
- The collective mode is fully separated from the continuum
- The upper mode acquires further weight and dispersion. Upper Hubbard band
- $\Delta \sim U$



$U/t = 0$: Particle-hole continuum ending with an edge peak

$U/t = 1$: A collective mode and an upper Hubbard band start to develop

$U/t = 4$: The collective mode separates from the continuum. The upper Hubbard band is clearly developed

$U/t = 12$: The upper Hubbard band is fully developed. $\Delta \simeq U$

V. H. Dao, RF (2016)

Summary and outlook

- ☞ The most prominent slave boson representations have been reviewed, from the SIAM to the Hubbard Model extended by $\mathbf{S_i} \cdot \mathbf{S_j}$ and $n_i n_j$ interactions.
- ☞ A path integral representation of a radial slave boson field on a discretized time mesh has been presented.
- ☞ This representation has been made use of to exactly evaluate the path integral for a toy model in the strong interaction limit.
- ☞ The low frequency-small momentum spin and charge response functions take an RPA form, with channel dependent effective interactions.
- ☞ At half-filling, F_0^a shows no singularity for $J = 0$ only.
- ☞ Ferromagnetic instabilities in a large part of the phase diagram for $J < -U/20$.
- ☞ Charge instabilities for $V < -0.234U$ at half-filling, and in a large part of the phase diagram for $V < -0.15U$.
- ☞ The Kotliar and Ruckenstein representation allows to describe the splitting of the charge excitation spectrum of the intermediate U Hubbard model.

Perspectives

- ☞ Finite $\vec{k}$ instabilities?
- ☞ Excitations of symmetry broken phases?

THANKS TO

- ☞ Peter Wölfle, Karlsruhe.
- ☞ Thilo Kopp, Augsburg.
- ☞ Andrzej M. Oleś, Kraków.
- ☞ Gabi Kotliar, Rutgers.
- ☞ Grégoire Lhoutellier, Caen-Rouen.
- ☞ Vu-Hung Dao, Caen.